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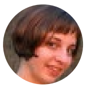

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Summary

In this paper, flexural strength, σ_{fs}, of cold isostatically pressed (CIP) high purity alumina ceramics was determined by the three-point bend test. It was found that flexural strength shows a large variation and can be described by a two-parameter Weibull distribution function, with the scale parameter σ_0, and the Weibull modulus m. These parameters were determined by the linear regression using the measured values of σ_{fs}. It was found that the measured flexural strength, σ_{fs}, of alumina ceramics follows the Weibull distribution. The flexural strength values were found to range from 266.7 to 357.5 MPa.

Key words: *flexural strength, alumina ceramics, Weibull distribution*

1. Introduction

Brittle materials, like ceramics, have numerous useful properties, such as high hardness, stiffness (modulus of elasticity), wear resistance, strength retention at elevated temperatures, corrosion resistance associated with chemical inertness, etc. Moreover, they have about 50% lower density than steel, which makes them suitable for all technical applications where weight reduction considerably reduces the consumption of energy [1-3]. The major limitations of ceramics for structural and specific non-structural applications are poor toughness and low strength reliability, which leads to expensive test procedures for the characterization of the material and a more difficult design in construction. Strength properties of ceramics are usually scattered, therefore, a statistical analysis is indispensable for the understanding of the mechanical characterization of this material.

Weibull statistics has been widely used to describe the statistical behaviour of mechanical properties of many materials, such as advanced ceramics, glass, metallic matrix composites, ceramic matrix composite and polymeric matrix composites [4-6].

In the present work, the flexural strength of cold isostatically pressed alumina ceramics, (CIP)-Al_2O_3, with a purity of 99.8 %, was analyzed by a two-parameter Weibull distribution.

2. Experiment

Experiments were performed on cold isostatically pressed (CIP)-Al_2O_3 with a purity of 99.8 %, supplied by Applied Ceramics, Inc., Fremont, California, U.S.A. Samples of alumina ceramics disks were sintered in a gas furnace at 1650 °C. The Archimedes density of the CIP-Al_2O_3 was 3.91 g/cm^3. The chemical composition of the investigated alumina ceramics, according to the manufacturer's declaration, is shown in Table 1. Al_2O_3 ceramics contains MgO as a sintering aid and the usual impurities, i.e. SiO_2, CaO, Na_2O and Fe_2O_3.

Table 1 Chemical composition of Al_2O_3 ceramics.

Sample	*Wt.* %					
	MgO	Fe_2O_3	SiO_2	Na_2O	CaO	Al_2O_3
Alumina ceramics	0.066	0.015	0.02	0.05	0.013	rest

After sintering, an alumina ceramics disk was cut into thirty-three rods. The test specimens (rods) were machined from the alumina ceramics disk by a diamond core drill. Each rod should have a radius of 5 mm and a length of 50 mm. The diameters of all test specimens are within the range of 5±0.02 mm and this variation did not influence the results of flexural strength. After machining, cylinders were thoroughly cleaned with alcohol in an ultrasound bath and dried in a sterilizer at 150 ± 5 ºC for 4 hours [7,8]. The specimens were then annealed at 1200 °C for 360 min in order to eliminate residual stresses that may have occurred during machining. The roughness of samples was measured by means of a Perthometer S&P (Feinprut Perthen GmbH, Goettingen, Germany). The average roughness (R_a) of alumina ceramics is 1.2 μm. The flexural strength of alumina ceramics at room temperature was measured using a three-point bending test configuration (Figure 1) [8]. The three-point bending tests were done using a MESSPHYSIK testing machine (BETA 50-5) for loading measurement and a videoextensometer (NG version 5.15.3) for deflection measurement. The applied load and deflection of the sample were recorded simultaneously.

3. Results and discussion

3.1. Three -point bending test

Due to extremely low ductility of ceramic materials, their mechanical properties cannot be measured by a conventional tensile test which is widely used for metals. Brittle materials, including ceramics, are tested by a flexure test (transverse beam test, bending test). There are two standard flexure test methods: a three-point test and a four-point test. In the three-point flexure test (Figure 1), a specimen with a round, rectangular or flat cross-section is placed on two parallel supporting pins. A loading force is applied in the middle by means of a loading pin. The supporting and loading pins are mounted in the way to allow their free rotation about the axis parallel to the pin axis. This configuration provides uniform loading of the specimen and prevents friction between the specimen and the supporting pins.

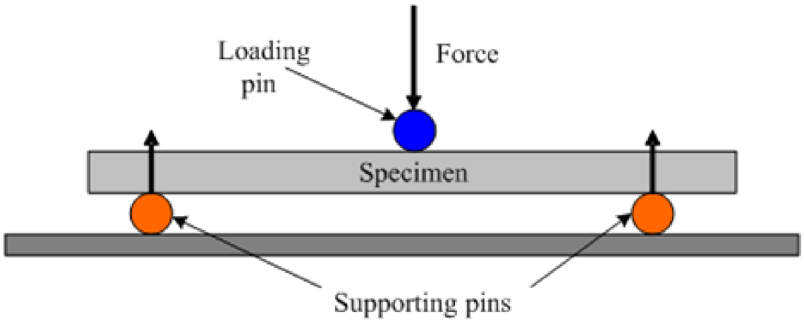

Fig. 1 Three-point bending test [8].

The stress at fracture which occured using this test is called flexural strength (σ_{fs}) or modulus of rupture (MOR) or fracture strength or bend strength. For samples with rectangular cross-sections, the fracture strength is given by:

$$\sigma_{fs} = \text{MOR} = F_f L / \pi R^3 \tag{1}$$

Where

- F_f is the load at fracture in N,
- L is the distance between supporting pins in mm,
- R is the specimen radius in mm.

Figure 2 shows results of the applied load and the deflection of thirty-three alumina ceramics samples.

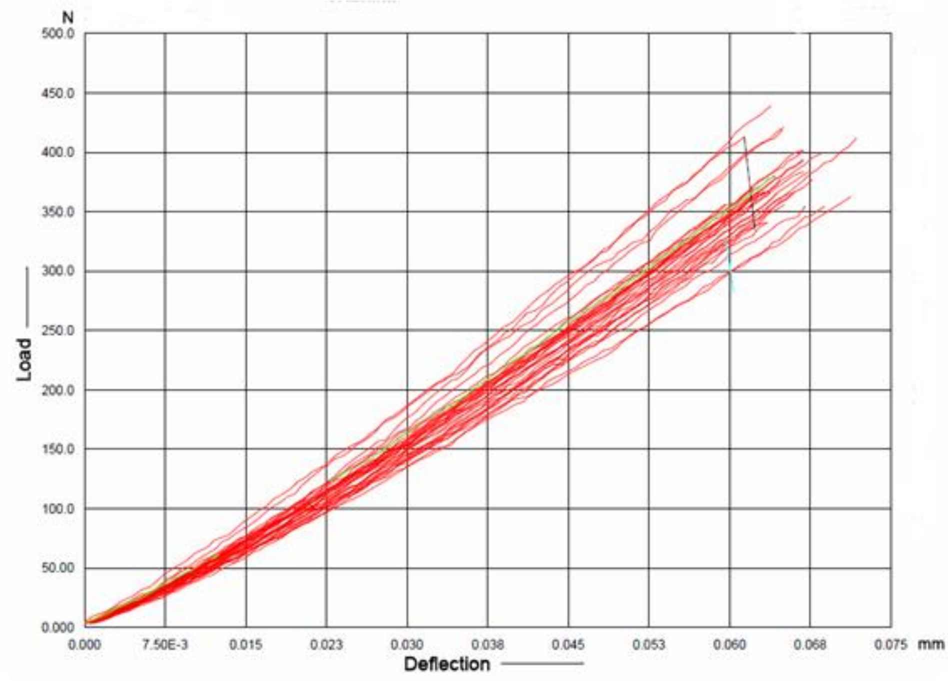

Fig. 2 Results of the applied load and the deflection of thirty-three samples of alumina ceramics.

3.2. The Weibull distribution

The Weibull distribution has been used successfully to describe a wide range of problems including the mechanical properties of brittle materials and life time testing [4-6]. The two-parameter continuous probability density function for strength variables is given by:

$$P=\left(\frac{m}{\sigma_0}\right)\left(\frac{\sigma}{\sigma_0}\right)^{m-1}\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right] \qquad (2)$$

The density function is asymmetrical about the mean and will assume only positive values. The symbol m is the Weibull modulus and σ_0 is the scale parameter. The density function for m with values of 1, 4 and 10 is illustrated in Figure 3A.

The cumulative distribution function that gives the probability of failure P at a stress σ is expressed as:

$$P=1-\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{m}\right] \qquad (3)$$

where P is the probability of failure at stress σ, m and σ_0 are the Weibull modulus and scale parameter, respectively.

This function corresponds to the exponential function for m=1 and has the sigmoid shape for m>1 (Figure 3B). The scale parameter σ_0 describes the stress level that causes failure in 63.2% of the specimens ($P(\sigma_0)=1-1/e=0.632$), m is the Weibull modulus which characterizes the width of the fracture distribution. The Weibull modulus or slope, m, is the measure of material homogeneity (variation of flexural strength). A high value of m indicates a high degree of homogeneity and a smaller variation in flexural strength (Figure 3).

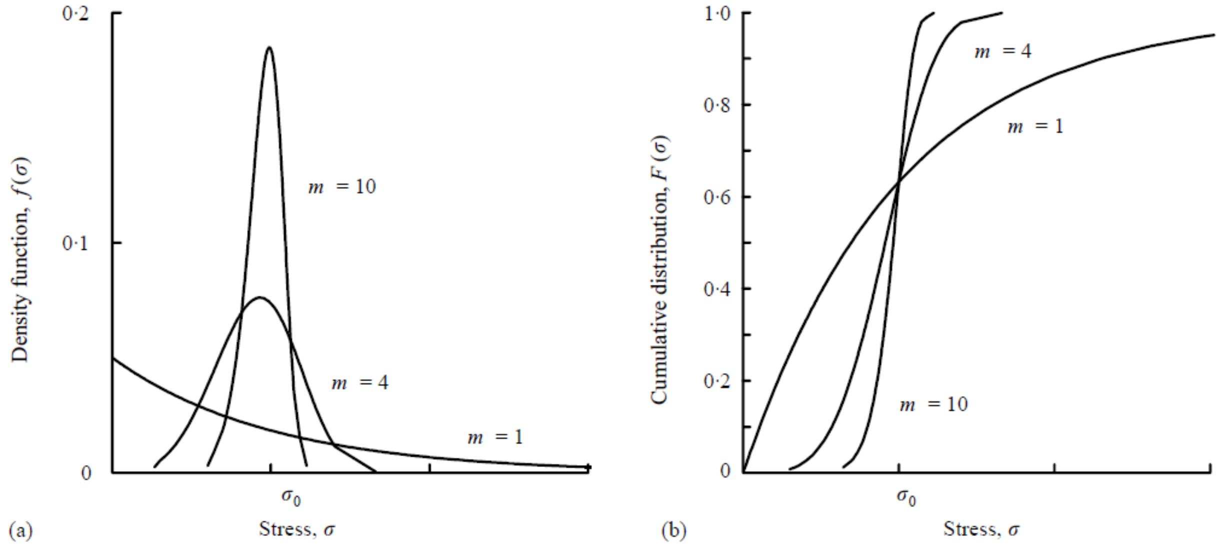

Fig. 3 (A) The Weibull probability density function (equation 1) and (B) the cumulative distribution function (equation 2), for three values of the Weibull modulus m (1, 4, 10) at a constant scale parameter σ_0 [2].

3.3. Estimation of Weibull parameters

Several procedures have been proposed for the determination of Weibull parameters, such as: linear regression (the Weibull plot), weighted linear regression and maximum likelihood [9-12]. The Weibull plot (linear-regression method) is the most common and easiest way to obtain Weibull parameters. The Weibull and the scale parameter are obtained by the linear regression analysis the following equation:

$$\ln \ln \left(\frac{1}{1-P} \right) = m \ln \sigma - m \ln \sigma_0 \quad (4)$$

From Figure 4, one can see that a straight line can be obtained by plotting $\ln \ln \left(\frac{1}{1-P} \right)$ against ln *σ*. The slope is *m* and the intercept is *m* ln σ_0. As it can be seen, the Weibull parameter *m* is 17.4 with a correlation coefficient of 0.9111. The scale parameter σ_0 is estimated from the intercept (*m* ln σ_0):

$$\sigma_0 = \exp\left(-(m \ln \sigma_0)/m\right) \quad (5)$$

The obtained values of *m* and σ_0 are inserted into equation (3), yielding the following:

$$P = 1 - \exp\left[-\left(\frac{\sigma}{316} \right)^{17.4} \right] \quad (6)$$

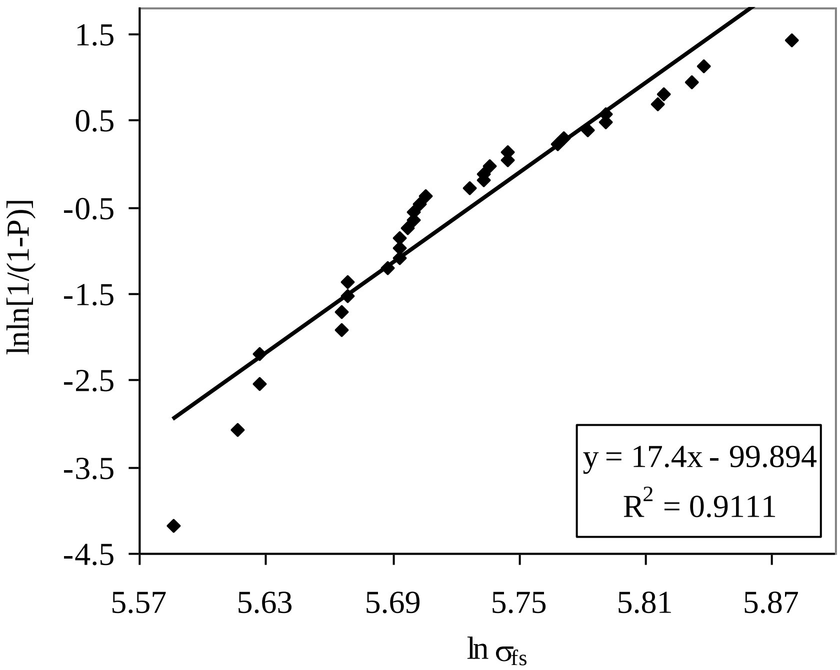

Fig. 4 Weibull plot for the flexural strength of cold isostatically pressed alumina ceramics, (CIP)-Al_2O_3).

Statistical analysis results of flexural strength (σ_{fs}) obtained for the thirty-three Al_2O_3 ceramics specimens, including the minimum, the maximum, the arithmetic average value and its standard deviation as well as Weibull parameters, are summarized in Table 2.

Table 2 Statistical analysis results for the measured flexural strength (σ_{fs}) of Al_2O_3 ceramics.

Properties	Minimum	Maximum	Average	Standard deviation	Weibull parameters	
					σ_0	m
σ_{fs}, MPa	266.7	357.5	306.9	21.37	316	17.4±0.97

Figure 5 shows a cumulative distribution function for the normalized flexural strength, σ_{fs}, of the cold isostatically pressed alumina ceramics, (CIP)-Al_2O_3. The flexural strength values are ranked from the minimum to the maximum value and each value is assigned a probability of failure (*P*) based on its ranking *i*, with *i* ranging from 1 to *n*, where *n* is the number of measurements of flexural strength (in this case *n* = 33). In the construction of this plot, the thirty-three measured σ data were normalized by their average value σ_{av}.

The cumulative probability of failure (*P*) is calculated using the following equation [9,12]:

$$P_i = \frac{i - 0.5}{n} \qquad (6)$$

where *i* is the rank and *n* is the total number of data. In this case, *n* is the total number of data on the measured fracture toughness. Other estimators, such as $P_i = \frac{i}{n+1}$; $P_i = \frac{i - 0.3}{n + 0.4}$

and $P_i = \frac{i - 3/8}{n + 1/4}$ have also been used [9-12]. The shape of the cumulative probability function is shown in Figure 5.

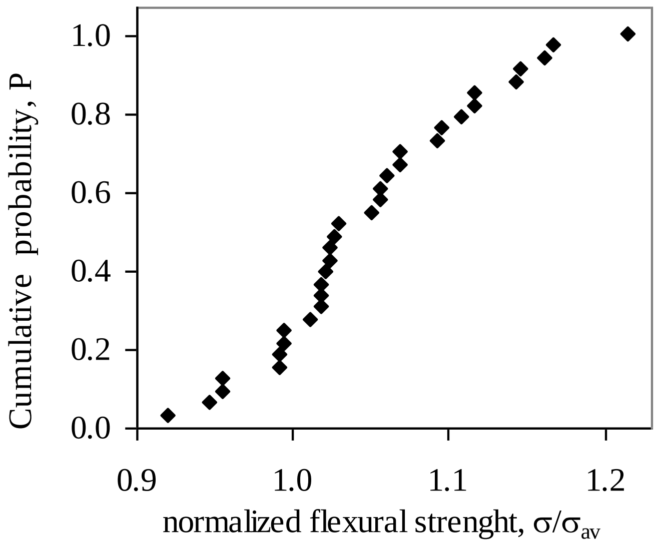

Fig. 5 Cumulative probability function

4. Conclusions

The flexural strength of high purity alumina ceramics was determined by the three-point bend test. The measured values of flexural strength were in the range from 266.7 to 357.5 MPa. It was found that a two-parameter Weibull distribution function may be employed to describe the statistical variability of flexural strength measured by the three-point bend test. The Weibull modulus (*m*) for flexural strength was found to be 17.4. This parameter can be used to describe the variability in the flexural strength as well as in the homogeneity of the tested material.

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