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ROBUST OUTPUT TRACKING CONTROL OF A QUADROTOR IN THE PRESENCE OF EXTERNAL DISTURBANCES

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Summary

In this paper, a robust output tracking controller for the quadrotor helicopter is proposed. The proposed controller requires the measurement of only four state variables: positions in the inertial coordinate frame and the yaw angle. Also, the controller is robust with respect to the unmodelled dynamics and provides rejections of all external force and torque disturbances. The effectiveness of the proposed controller is tested on a simulation example of a quadrotor tracking under the influence of wind which is modelled as unmatched external force disturbances in the horizontal plane.

Key words: *quadrotor, tracking control, robust control, output control*

1. Introduction

The quadrotor helicopter is a small agile vehicle controlled by four rotors. Compared with other flying vehicles, quadrotors have specific characteristics that allow the performance of applications that would be difficult or impossible otherwise. This superiority is due to their unique ability to carry out vertical, stationary and low speed flight. The quadrotor also has a higher payload capacity compared to conventional flying vehicles. The main disadvantage of the quadrotor is its high energy requirement because it uses four motors.

From the control point of view, the quadrotor is a highly nonlinear, multivariable, strongly coupled and underactuated system which has six degrees of freedom and only four actuators. Its low cost and simple mechanical structure are additional advantages meaning that the quadrotor provides an excellent testing ground for the application of advanced control techniques.

Various advanced control methods have been developed, such as feedback linearization method [1, 2], adaptive control [3, 4], sliding-mode control [5], backstepping control [6], H_∞ robust control [7], etc. However, most of the proposed methods require full information on the state that may limit their practical applicability, because the increased number in sensors makes the overall system more complex in implementation and expensive in realization.

Several solutions have been proposed with the aim of reducing the number of sensors. In [8], the authors proposed a velocity estimator for the tracking control of an under-actuated quadrotor using only linear and angular positions. In [9] and [10], sliding-mode observers were proposed to estimate the effect of external perturbations using measurement of positions and yaw angle.

The main practical difficulties of the quadrotor tracking control are parametric uncertainties, unmodelled dynamics, and external disturbances. During quadrotor flights, sudden wind gusts can significantly affect flight performance and even cause instability [11, 12]. In order to accomplish high level tracking performances, robust flight control systems are required to track desired trajectories in the presence of wind or other disturbances. Recently, several solutions have been proposed for reducing the impact of wind disturbances on quadrotor tracking performances, mostly based on sliding-mode disturbance observers [13, 14]. The proposed control laws are based on the full-state measurement, which requires a large number of sensors.

Further, the outer feedback loop is based on the feedback linearization of full translational dynamics of the quadrotor, leading to computationally expensive control laws.

In this paper, we propose a robust output tracking controller which requires the measurement of the quadrotor's linear position and yaw angle only. The proposed controller provides the rejection of external forces and torque disturbances, such as wind gusts. The control design is based on a bilinear reduced model of the quadrotor, which preserves the relative degree of the system. The resulting outer control-loop, which is based on the feedback linearization, has a very simple structure compared to the controllers described in literature. The inner control-loop is based on the sliding-mode control design with the aim of compensating for all disturbances and unmodelled dynamics. Further, sliding-mode filters are used to estimate all derivatives in the control law providing a significant reduction in the required number of sensors. Also, a smoothing nonlinear filter is used to prevent sudden jumps of the control variables.

The paper is organized as follows. In Section 2, the dynamic model of the quadrotor helicopter is developed. Based on this nonlinear dynamic model, Section 3 deals with the design of a robust output tracking controller. The simulation results are presented in Section 4. Finally, concluding remarks are emphasized in Section 5.

2. Quadrotor dynamic model

The quadrotor helicopter is made of a rigid cross frame equipped with four rotors. The equations describing the altitude and the attitude motions of the quadrotor helicopter are basically the same as those describing a rotating rigid body with six degrees of freedom [15, 16].

2.1 Quadrotor attitude dynamics and kinematics

The three-axis rotational dynamics of a rigid body in the body-fixed reference frame is given by

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega}) = \boldsymbol{\tau} + \mathbf{d}_\tau \tag{1}$$

where $\mathbf{I} = \mathrm{diag}\{I_x, I_y, I_z\}$ is the diagonal inertia matrix, $\boldsymbol{\omega} = [p\ q\ r]^{\mathrm{T}}$ is the angular velocity vector, $\boldsymbol{\tau} = [\tau_\phi\ \tau_\theta\ \tau_\psi]^{\mathrm{T}}$ is the vector of actuator torques, and $\mathbf{d}_\tau = [d_\phi\ d_\theta\ d_\psi]^{\mathrm{T}}$ is the vector of external disturbance torques.

The rigid body rotational kinematics equations are given by

$$\dot{\boldsymbol{\eta}} = \boldsymbol{\Omega}_B \boldsymbol{\omega} \tag{2}$$

where $\boldsymbol{\eta} = [\phi\ \theta\ \psi]^{\mathrm{T}}$ are the Euler angles defined according to the *xyz*-convention, and

$$\mathbf{\Omega}_B = \frac{1}{c_\theta}\begin{bmatrix} c_\theta & s_\phi s_\theta & c_\phi s_\theta \\ 0 & c_\phi c_\theta & -s_\phi c_\theta \\ 0 & s_\phi & c_\phi \end{bmatrix} \tag{3}$$

is the transformation matrix from the body to inertial coordinate frame, where $c_\eta \equiv \cos(\eta)$ and $s_\eta \equiv \sin(\eta)$.

2.2 Quadrotor translational dynamics and kinematics

The translational dynamics model of the rigid body in the body-fixed reference frame is given by

$$m\dot{\mathbf{v}} + \boldsymbol{\omega} \times (m\mathbf{v}) = \mathbf{F} + \mathbf{d}_F \tag{4}$$

where m is the mass of the quadrotor, $\mathbf{F} = [F_x\ F_y\ F_z]^\mathrm{T}$ is the vector of external forces, $\mathbf{v} = [u\ v\ w]^\mathrm{T}$ is the quadrotor linear velocity, and $\mathbf{d}_F = [d_x\ d_y\ d_z]^\mathrm{T}$ is the vector of external disturbance forces.

The rigid body translational kinematics equations are given by

$$\dot{\boldsymbol{\xi}} = \mathbf{R}\mathbf{v} \tag{5}$$

where $\boldsymbol{\xi} = [x\ y\ z]^\mathrm{T}$ is the vector of translational positions in the inertial coordinate frame, and R is the rotation matrix given by

$$\mathbf{R} = \mathbf{R}_z^T(\psi)\mathbf{R}_y^T(\theta)\mathbf{R}_x^T(\phi) \tag{6}$$

where particular rotational matrices are

$$\mathbf{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{bmatrix}, \quad \mathbf{R}_y(\theta) = \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix}, \quad \mathbf{R}_z(\psi) = \begin{bmatrix} c_\psi & s_\psi & 0 \\ -s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

2.3 Quadrotor forces and moments

The four propellers of the quadrotor, rotating at angular velocities Ω_i produce four forces

$$f_i = k_i \Omega_i^2 \tag{8}$$

directed upwards, where i=1, 2, 3, 4, and k_i are positive constants. The use of the four forces f_i as input to the system is somewhat counterintuitive. Thus, it is common to use a control allocation scheme that transforms the forces f_i into a vertical thrust $\mathbf{F} = [0\ 0\ F_z]^\mathrm{T}$ and three torques around the three orthogonal body axes τ_ϕ, τ_θ, τ_ψ.

Thus, the control distribution from the four actuator motors of the quadrotor is given by

$$\begin{bmatrix} F_z \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -l & 0 & l & 0 \\ 0 & l & 0 & -l \\ c & -c & c & -c \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{9}$$

where l is the distance from the motors to the centre of gravity, and c is a constant known as the force-to-moment scaling factor. So, if the required thrust and torque vector are given then the rotor force can be calculated by using (9).

2.4 Quadrotor forces and moments

During the quadrotor navigation with a moderate velocity, the roll and pitch angles remain near zero degrees to allow the approximation of matrix $\mathbf{\Omega}_B$ with the identity matrix, thus the vector of derivation of the Euler angles $\dot{\boldsymbol{\eta}}$ can be approximated by the body axis angular velocity $\boldsymbol{\omega}$. Under these assumptions, the dynamic model of the quadrotor (1)-(8) can be reduced to a more appropriate form for the control system design

$$\begin{aligned} m\ddot{x} &= \left(c_\psi s_\theta c_\phi + s_\psi s_\phi\right)u_1 + d_x \\ m\ddot{y} &= \left(s_\psi s_\theta c_\phi - c_\psi s_\phi\right)u_1 + d_y \\ m\ddot{z} &= -mg + \left(c_\phi c_\theta\right)u_1 + d_z \end{aligned} \tag{10}$$

$$\begin{aligned} I_x\ddot{\phi} &= \left(I_y - I_z\right)\dot{\theta}\dot{\psi} + u_2 + d_\phi \\ I_y\ddot{\theta} &= \left(I_z - I_x\right)\dot{\phi}\dot{\psi} + u_3 + d_\theta \\ I_z\ddot{\psi} &= \left(I_x - I_y\right)\dot{\phi}\dot{\theta} + u_4 + d_\psi \end{aligned} \tag{11}$$

where

$$u_1 = F_z,\ \ u_2 = \tau_\phi,\ \ u_3 = \tau_\theta,\ \ u_4 = \tau_\psi \tag{12}$$

are control variables, and d_x, d_y, d_z, d_ϕ, d_θ, d_ψ are external disturbances.

3. Output tracking control

The quadrotor is an underactuated system, because six state variables $\{x, y, z, \phi, \theta, \psi\}$ should be controlled by only four inputs $\{F_z, \tau_\phi, \tau_\theta, \tau_\psi\}$. Therefore, in most cases, only some of the system variables, for example $\{x, y, y, z, \psi\}$, are usually controlled.

The proposed robust output controller synthesis has three main phases: a) feedback linearization; b) sliding-mode control design; c) sliding-mode derivative filters design.

3.1 Feedback linearization controller (FLC) design

The feedback linearization of the control design on the model (10), (11) leads to an extremely complicated control law, as can be seen in [10]. But, if the feedback linearization is just the first step of the robust sliding-mode controller design, we can use an additional simplification of the model (10), (11) which preserves relative degrees of the system. Under the assumption of small angles η_i, i=1, 2, 3, follows that cos(η_i) ≈ 1 and sin(η_i) ≈ η_i. Further, when neglecting the quadratic terms, the model (10), (11) becomes

$$\begin{aligned} m\ddot{x} &= \theta u_1 + d_{mx} \\ m\ddot{y} &= -\phi u_1 + d_{my} \\ m\ddot{z} &= -mg + u_1 + d_{mz} \end{aligned} \tag{13}$$

$$\begin{aligned} I_x\ddot{\phi} &= u_2 + d_{m\phi} \\ I_y\ddot{\theta} &= u_3 + d_{m\theta} \\ I_z\ddot{\psi} &= u_4 + d_{m\psi} \end{aligned} \tag{14}$$

which is a bilinear system with the same relative degrees as the system (10), (11). Disturbances d_{mx}, d_{my}, d_{mz}, $d_{m\phi}$, $d_{m\theta}$, $d_{m\psi}$ represent external disturbances and modelling errors due to the simplification of the original system (10), (11).

The control goal is the tracking of the desired quadrotor position $\{x_d\,(t), y_d\,(t), z_d\,(t)\}$, preserving angle ψ_d = 0, in the presence of unknown external disturbances.

So, the tracking error variables are

$$\tilde{x} = x - x_d(t), \quad \tilde{y} = y - y_d(t), \quad \tilde{z} = z - z_d(t) \tag{15}$$

The first step is to design the control law for u_1 which will stabilize the altitude tracking error

$$m\ddot{\tilde{z}} = -m\ddot{z}_d - mg + u_1 + d_{mz} \tag{16}$$

The choice of the control variable

$$u_1 = m\left(\ddot{z}_d + g - k_{z1}\dot{\tilde{z}} - k_{z0}\tilde{z}\right) + u_{1s} \tag{17}$$

leads to the error dynamics

$$\ddot{\tilde{z}} + k_{z1}\dot{\tilde{z}} + k_{z0}\tilde{z} = \frac{1}{m}\left(u_{1s} + d_{mz}\right) \tag{18}$$

where k_{z1} and k_{z0} are positive gains, and u_{1s} is the sliding-mode part of the controller which will be designed to compensate for unknown disturbance d_{mz}. In that case, the right side of (18) as well as the altitude error converge to zero for k_{z1}, $k_{z0} > 0$.

The tracking error $\tilde{x}$ will be stabilized through the control variable u_3, since the first equation in (13) depends only on angle θ. The second order error equation is

$$m\ddot{\tilde{x}} = -m\ddot{x}_d + \theta u_1 + d_{mx} \tag{19}$$

The equation (19) should be derived twice to provide an explicit appearance of control variable u_3. So, the first derivation of (19) is

$$m\dddot{\tilde{x}} = -m\dddot{x}_d + \dot{\theta}u_1 + \theta\dot{u}_1 + \dot{d}_{mx} \tag{20}$$

and the second derivation is

$$m\tilde{x}^{(4)} = -mx_d^{(4)} + \ddot{\theta}u_1 + 2\dot{\theta}\dot{u}_1 + \theta\ddot{u}_1 + \ddot{d}_{mx} \tag{21}$$

Inserting the second equation of (14) in (21), we get

$$m\tilde{x}^{(4)} = -mx_d^{(4)} + u_1\frac{u_3 + d_{m\theta}}{I_y} + 2\dot{\theta}\dot{u}_1 + \theta\ddot{u}_1 + \ddot{d}_{mx} \tag{22}$$

The choice of the control variable

$$u_3 = \frac{I_y}{u_1}\left(mx_d^{(4)} - 2\dot{\theta}\dot{u}_1 - \theta\ddot{u}_1 + mu_x\right) + u_{3s} \tag{23}$$

where

$$u_x = -k_{x3}\dddot{\tilde{x}} - k_{x2}\ddot{\tilde{x}} - k_{x1}\dot{\tilde{x}} - k_{x0}\tilde{x} \tag{24}$$

leads to the error dynamics

$$\tilde{x}^{(4)} + k_{x3}\dddot{\tilde{x}} + k_{x2}\ddot{\tilde{x}} + k_{x1}\dot{\tilde{x}} + k_{x0}\tilde{x} = \frac{u_1}{mI_y}\left(u_{3s} + d_{cx}\right) \tag{25}$$

where k_{x3}, k_{x2}, k_{x1} and k_{x0} are positive gains that satisfy the Hurwitz stability condition, and u_{3s} is the sliding-mode part of the controller, which will be designed to compensate for cumulative disturbance d_{cx}

$$d_{cx} = \frac{I_y}{u_1}\ddot{d}_{mx} + d_{m\theta} \tag{26}$$

The tracking error $\tilde{y}$ will be stabilized through the control variable u_2, since the second equation in (13) depends only on angle ϕ. The second, third and fourth order error equations are

$$m\ddot{\tilde{y}} = -m\ddot{y}_d - \phi u_1 + d_{my} \tag{27}$$

$$m\dddot{\tilde{y}} = -m\dddot{y}_d - \dot{\phi} u_1 - \phi\dot{u}_1 + \dot{d}_{my} \tag{28}$$

$$m\tilde{y}^{(4)} = -my_d^{(4)} - u_1\frac{u_2 + d_{m\phi}}{I_x} - 2\dot{\phi}\dot{u}_1 - \phi\ddot{u}_1 + \ddot{d}_{my} \tag{29}$$

The choice of the control variable

$$u_2 = -\frac{I_x}{u_1}\left(my_d^{(4)} - 2\dot{\phi}\dot{u}_1 - \phi\ddot{u}_1 + mu_y\right) - u_{2s} \tag{30}$$

where

$$u_y = -k_{y3}\dddot{\tilde{y}} - k_{y2}\ddot{\tilde{y}} - k_{y1}\dot{\tilde{y}} - k_{y0}\tilde{y} \tag{31}$$

leads to the error dynamics

$$\tilde{y}^{(4)} + k_{y3}\dddot{\tilde{y}} + k_{y2}\ddot{\tilde{y}} + k_{y1}\dot{\tilde{y}} + k_{y0}\tilde{y} = \frac{u_1}{mI_x}\left(u_{2s} + d_{cy}\right) \tag{32}$$

where k_{y3}, k_{y2}, k_{y1} and k_{y0} are positive gains that satisfy the Hurwitz stability conditions, and u_{2s} is the sliding-mode part of the controller, which will be designed to compensate for cumulative disturbance d_{cy}

$$d_{cy} = \frac{I_x}{u_1}\ddot{d}_{my} - d_{m\phi} \tag{33}$$

Finally, the angle ψ can be stabilized by a PD controller

$$u_4 = -I_z k_{\psi 1}\dot{\psi} - I_z k_{\psi 0}\psi \tag{34}$$

so that the error dynamics has the following form

$$\ddot{\psi} + k_{\psi 1}\dot{\psi} + k_{\psi 0}\psi = \frac{1}{I_z}\left(u_{4s} + d_{m\psi}\right) \tag{35}$$

which is asymptotically stable for $k_{\psi 1}$, $k_{\psi 0} > 0$. Variable u_{4s} is the sliding-mode part of the controller which will be designed to compensate for unknown disturbance d_{mz}.

It can be seen that the overall feedback linearization controller (FLC) is a full-state controller, which means that the measurement of all 12 state variables is required. Also, the FLC cannot be applied in the presence of unmatched external disturbances, since the second and third error derivatives (19), (20), (27), (28), which are necessary for controllers (24) and (31), contain unknown disturbances d_{mx}, d_{my}, and their first derivations.

Note that the proposed control laws can guarantee convergent tracking only if control variable u_1 is strictly positive $u_1>0$, which can be achieved by including a saturation function in the control law (17)

$$u_1 = mg + k_g \tanh\left(\frac{m}{k_g}\left(\ddot{z}_d - k_{z1}\dot{\tilde{z}} - k_{z0}\tilde{z}\right) + \frac{1}{k_g}u_{1s}\right) \tag{36}$$

where positive parameter k_g satisfies the condition: $k_g<mg$.

3.2 Sliding-mode controller (SMC) design

The sliding-mode control laws are chosen to compensate for external and unmodelled disturbances. The error dynamics (18) can be represented as

$$\dot{s}_1 + \lambda_1 s_1 = \frac{1}{m}\left(u_{1s} + d_{mz}\right) \tag{37}$$

where the sliding variable s_1 has the following form

$$s_1 = \dot{\tilde{z}} + \alpha_{z0}\tilde{z} \tag{38}$$

Positive parameters λ_1 and α_{z0} satisfy the following conditions

$$k_{z1} = \alpha_{z0} + \lambda_1, \qquad k_{z0} = \alpha_{z0}\lambda_1 \tag{39}$$

The sliding-mode controller u_{1s} will be chosen as

$$u_{1s} = -\rho_1 \operatorname{sign}\left(s_1\right) = -\rho_1 \frac{s_1}{|s_1|} \tag{40}$$

where ρ_1 is a positive parameter which will be defined by using the Lyapunov stability analysis.

Taking the Lyapunov function

$$V_1 = \frac{1}{2}s_1^2 \tag{41}$$

we get time derivation

$$\dot{V}_1 = s_1\dot{s}_1 = s_1\left(-\lambda_1 s_1 + \frac{1}{m}\left(-\rho_1\frac{s_1}{|s_1|} + d_{mz}\right)\right) \tag{42}$$

where we used (37) and (40), so that

$$\dot{V}_1 = -\lambda_1 s_1^2 - \frac{\rho_1}{m}|s_1| + \frac{d_{mz}}{m}s_1 \le -\lambda_1 s_1^2 - \frac{1}{m}\left(\rho_1 - \max\left\{|d_{mz}|\right\}\right)|s_1| \tag{43}$$

The time derivation of the Lyapunov function will be negative definite if the following condition is satisfied

$$\rho_1 > \max\left\{|d_{mz}|\right\} \tag{44}$$

A similar approach can be applied to the second-order error equation (35). The sliding-mode controller u_{4s} will be chosen as

$$u_{4s} = -\rho_4 \operatorname{sign}\left(s_4\right) \tag{45}$$

where sliding variable s_4 has the following form

$$s_4 = \dot{\psi} + \alpha_{\psi 0}\psi \tag{46}$$

so that error equation (35) becomes

$$\dot{s}_4 + \lambda_4 s_4 = \frac{1}{I_z}\left(u_{4s} + d_{m\psi}\right) \tag{47}$$

Positive parameters λ_4, $\alpha_{\psi 0}$ and ρ_4 satisfy the following conditions

$$k_{\psi 1} = \alpha_{\psi 0} + \lambda_4, \quad k_{\psi 0} = \alpha_{\psi 0}\lambda_4, \quad \rho_4 > \max\left\{|d_{m\psi}|\right\} \tag{48}$$

The fourth-order error equation (25) can be represented as

$$\dot{s}_3 + \lambda_3 s_3 = \frac{u_1}{mI_y}\left(u_{3s} + d_{cx}\right) \tag{49}$$

where the sliding variable s_3 has the following form

$$s_3 = \dddot{\tilde{x}} + \alpha_{x2}\ddot{\tilde{x}} + \alpha_{x1}\dot{\tilde{x}} + \alpha_{x0}\tilde{x} \tag{50}$$

Positive parameters λ_3, α_{x2}, α_{x1} and α_{x0} satisfy the following conditions

$$k_{x3} = \alpha_{x2} + \lambda_3, \qquad k_{x2} = \alpha_{x1} + \lambda_3\alpha_{x2}, \qquad k_{x1} = \alpha_{x0} + \lambda_3\alpha_{x1}, \qquad k_{x0} = \lambda_3\alpha_{x0} \tag{51}$$

including the Hurwitz stability condition: $\alpha_{x2}\alpha_{x1} > \alpha_{x0}$. The sliding-mode controller u_{1s} will be chosen as

$$u_{3s} = -\rho_3 \operatorname{sign}\left(s_3\right) \tag{52}$$

where the positive parameter ρ_3 satisfies the following condition

$$\rho_3 > \max\left\{\left|d_{cx}\right|\right\} \tag{53}$$

A similar sliding-mode control law can be derived for the fourth-order error dynamics (32)

$$u_{2s} = -\rho_2 \operatorname{sign}\left(s_2\right), \qquad s_2 = \dddot{\tilde{y}} + \alpha_{y2}\ddot{\tilde{y}} + \alpha_{y1}\dot{\tilde{y}} + \alpha_{y0}\tilde{y} \tag{54}$$

where positive parameters ρ_2, α_{y2}, α_{y1} and α_{y0} satisfy similar conditions as (51) and (53).

The proposed sliding-mode controllers provide effective compensation for matched uncertainties d_{mz}, $d_{m\phi}$, $d_{m\theta}$, $d_{m\psi}$, but cannot provide compensation for unmatched uncertainties d_{mx}, d_{my}, since the higher-order error derivatives contain unknown disturbances and their first derivatives. This problem can be avoided by introducing filters in the form of sliding-mode derivative estimators.

3.3 Sliding-mode filter (SMF) design

One of the approaches to the estimation of time derivations of some continuous function $\zeta(t)$, is using the sliding-mode filter (SMF)

$$\dot{\zeta}_e = v_\zeta = -\rho_\zeta \operatorname{sign}\left(\zeta_e - \zeta\right) \tag{55}$$

where ζ_e is the estimation of ζ, and v_ζ is the estimation of $\dot{\zeta}$. The gain ρ_ζ is tuned to satisfy the following condition

$$\rho_\zeta > \max\left(\left|\dot{\zeta}\right|\right) \tag{56}$$

We use the SMF (55) to replace all time derivations in the proposed control laws with their estimates. Input functions of the SMF are

$$\boldsymbol{\zeta} = \begin{bmatrix} \tilde{x} & \tilde{y} & \tilde{z} & \dot{\tilde{x}}_e & \dot{\tilde{y}}_e & \ddot{\tilde{x}}_e & \ddot{\tilde{y}}_e & u_1 & \dot{u}_{1,e} & \psi \end{bmatrix}^T \tag{57}$$

and output derivation estimates are

$$\dot{\boldsymbol{\zeta}}_e = \begin{bmatrix} \dot{\tilde{x}}_e & \dot{\tilde{y}}_e & \dot{\tilde{z}}_e & \ddot{\tilde{x}}_e & \ddot{\tilde{y}}_e & \dddot{\tilde{x}}_e & \dddot{\tilde{y}}_e & \dot{u}_{1,e} & \ddot{u}_{1,e} & \dot{\psi}_e \end{bmatrix}^T \tag{58}$$

In this way, we get an output controller which requires the measurement of only four output variables $\{x, y, z, \psi\}$, and also provides compensation for unmatched disturbances d_{mx}, d_{my}.

Note that the signum function, which appears in the sliding-mode controllers and filters, can cause the chattering phenomena, or high-frequency oscillations of control variables. This problem can be avoided by replacing the discontinuous signum function with an appropriate continuous approximation, such as

$$\mathrm{sign}(\zeta) \approx \tanh(K\zeta) \tag{59}$$

where positive scalar K has some enough large value.

The initial conditions of the filters are set to zero, leading to large jumps in the control variables at the beginning of control actions. A common approach to preventing this problem is the filtration of the desired reference trajectory through a linear six-order filter [9]. But, this filtration significantly deforms the initial reference trajectory. In this paper, we use a saturated nonlinear smoothing filter, which preserves the initial form of the reference trajectory,

$$\dot{u}_{i,s} = -\rho_{u,i}\tanh\left(\mu_i(u_{i,s} - u_i)\right) \tag{60}$$

where $u_{i,s}$ is a filtered form of the control variable u_i, and μ_i is a positive scalar. The gain $\rho_{u,i}$ is tuned to satisfy the following condition

$$\rho_{u,i} > \max\left(|\dot{u}_i|\right) \tag{61}$$

So, by using gain $\rho_{u,i}$ we can reduce the maximum control slope.

4. Simulation results

In order to verify the effectiveness of the proposed control law, the overall system is tested in numerical simulations by using the Runge-Kutta's method with a variable time step. The physical parameters for the quadrotor are:

I_x = 0.62 N m s^2, I_y = 0.62 N m s^2, I_z = 1.24 N m s^2, m = 1 kg, g = 9.81 m/s^2.

The reference trajectory chosen for $x_d(t)$, $y_d(t)$, $z_d(t)$ and $\psi_d(t)$ is

$$x_d(t) = \cos(0.5t), \quad y_d(t) = \sin(0.5t), \quad z_d(t) = 0.5t, \quad \psi_d(t) = 0 \tag{62}$$

The initial conditions are: $x_d(0)$ = 0.5 m, $y_d(0)$ = 0 m, $z_d(0)$ = 0 m and $\psi_d(0)$ = 0 rad. All other initial conditions are zero.

To test the robustness of the controller, external disturbances have been introduced. The most likely disturbance acting on the quadrotor is wind in the horizontal plane, which can be modelled by forces d_{mx}, d_{my}, chosen as

$$d_{mx}(t) = 1.5 + 2.5\sin(4t), \qquad d_{my}(t) = 2.5 + 1.5\sin(3t) \tag{63}$$

All other external disturbances are set to zero.

The FLC+SMC gains for u_1 and u_4 are chosen as: λ_1=3, α_{z0}=3 and ρ_1=0. The FLC+SMC gains for u_2 and u_3 are chosen as: λ_3=3, α_{x2}=9, α_{x1}=27, α_{x0}=27 and ρ_3=4. The SMF gains, which appear in (55) and (59), are ρ_ζ=40 and K=20. The smoothing filter gains are $\rho_{u,i}$=80 and μ_i=5.

In Figures 1-4, we can see simulation results for the FLC+SMF in the case without external disturbances. It can be seen that the proposed controller provides practical exponential tracking [17], or almost asymptotic convergence toward reference trajectory. The small steady-state error, which can be seen in Fig. 2, is a consequence of the quadrotor model simplification. Such a small error and small values of the roll and pitch angle, which can be seen in Fig. 1, justify the starting assumptions for the synthesis of the FLC.

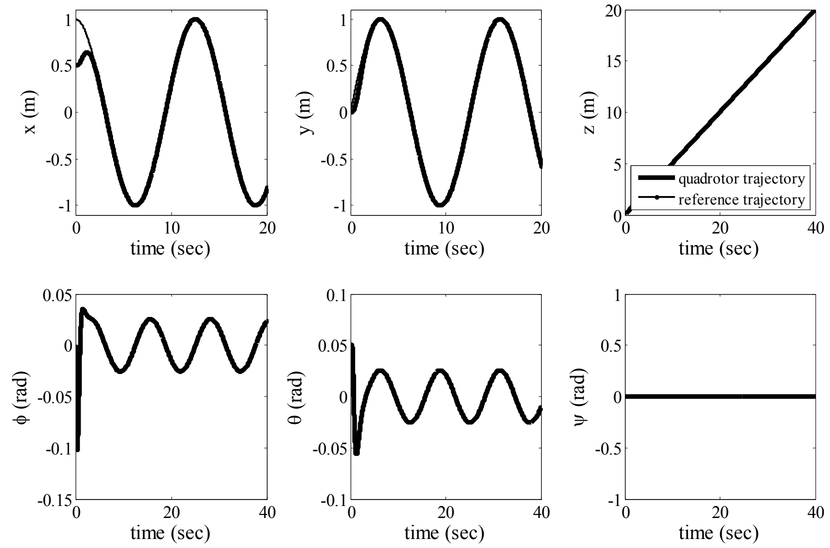

Fig. 1 The position and attitude of quadrotor in the closed-loop with FLC + SMF (the case without external disturbances)

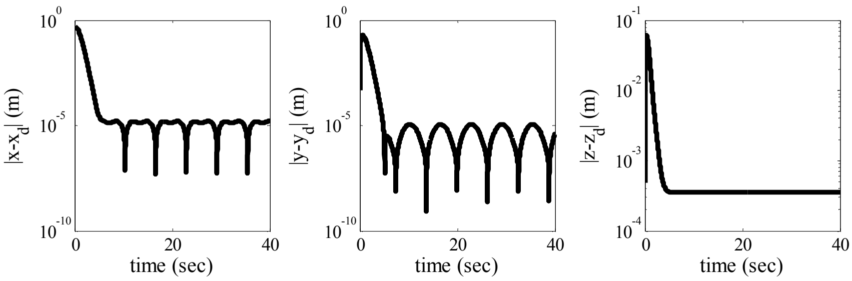

Fig. 2 The position errors of quadrotor in the closed-loop with FLC + SMF (the case without external disturbances)

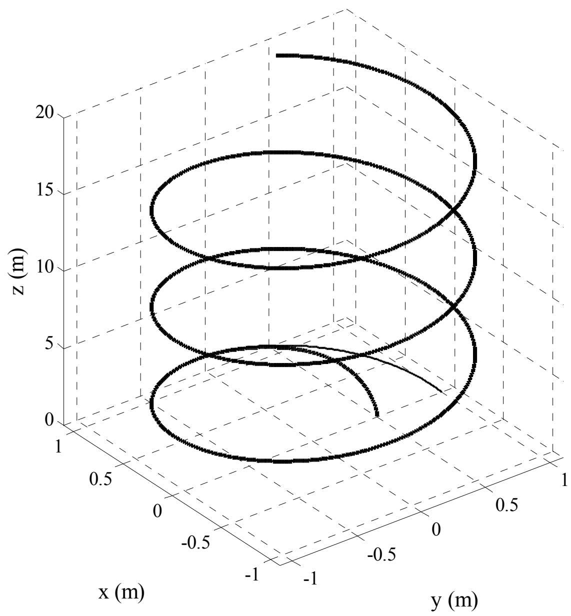

Fig. 3 The quadrotor and reference trajectory for case with FLC + SMF, without external disturbances

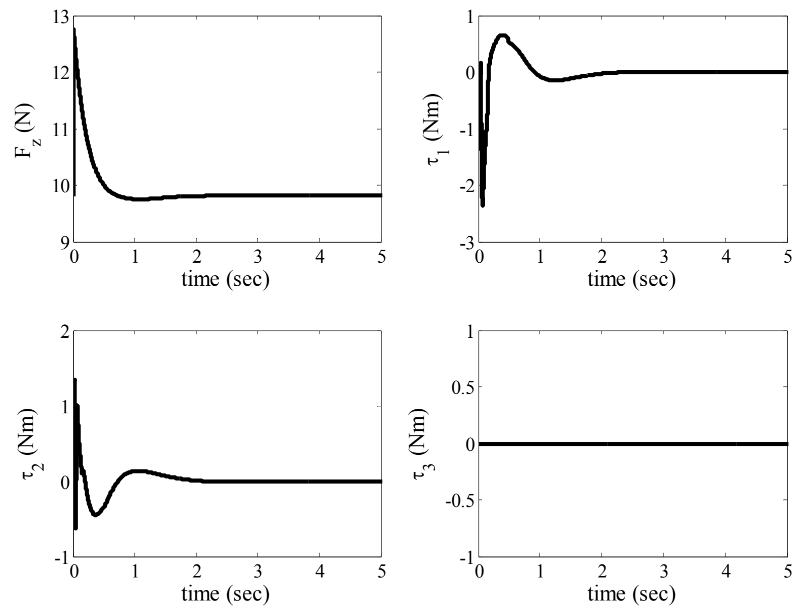

Fig. 4 The force and torques of quadrotor in the closed-loop with FLC + SMF (the case without external disturbances)

In Fig. 5, we can see simulation results for the FLC combined with the SMF in the case of external force disturbances. It can be seen that the FLC with the SMF cannot provide asymptotic tracking of reference trajectory. Fig. 6 shows simulation results for the FLC combined with the SMF and the SMC in the presence of external disturbances. It can be seen that the sliding-mode part of the controller ensures efficient disturbance rejection, providing convergent tracking of reference trajectory.

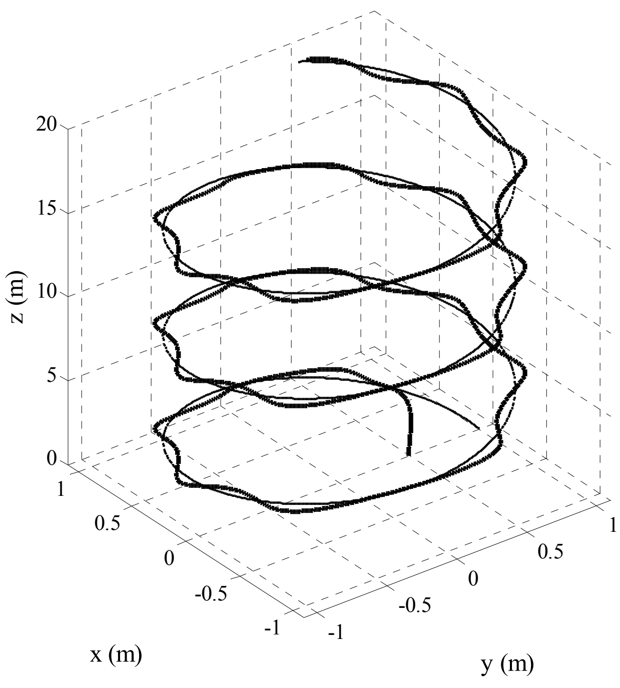

Fig. 5 The quadrotor and reference trajectory for case with FLC + SMF, with external disturbances

In Fig. 4 and Fig. 7 we can see forces and torques for the FLC + SMF in the case without external disturbances, and for the FLC + SMC + SMF in the case with external disturbances, respectively. In both cases there are no control jumps which appear in the case without smoothing filters (60).

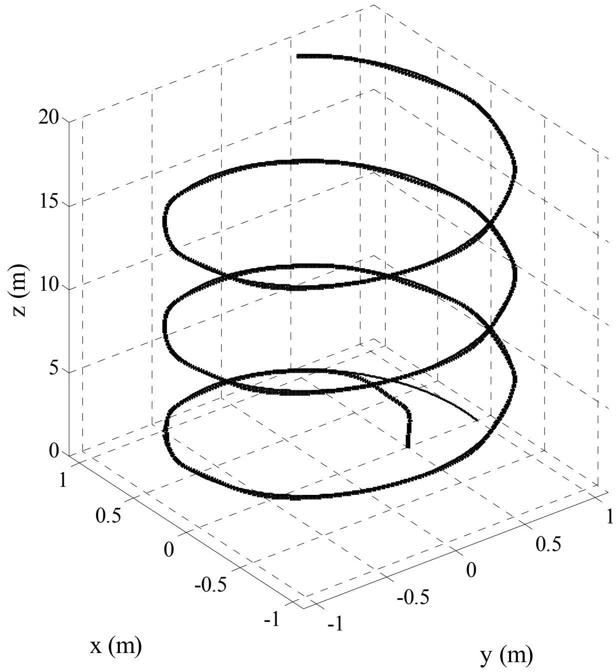

Fig. 6 The quadrotor and reference trajectory for case with FLC + SMC + SMF, with external disturbances

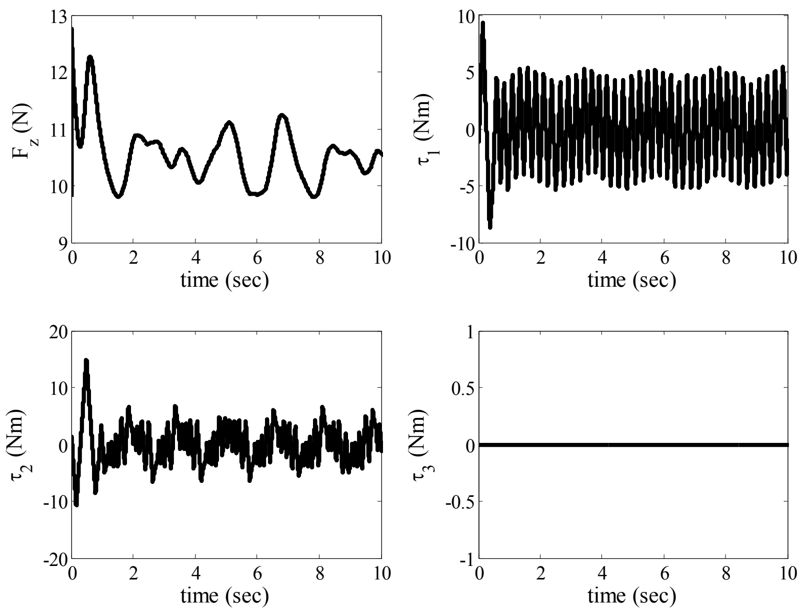

Fig. 7 The force and torques of quadrotor in the closed-loop with FLC + SMC + SMF, with external disturbances

Simulation results for other choices of reference trajectories and initial conditions show similar behavior. The controller also shows high robustness with respect to a change in the system parameters.

5. Conclusions

In this paper, performances of a robust output controller are demonstrated on a challenging problem: quadrotor tracking under unmatched external force disturbances in the horizontal plane. The simulation results illustrate that the proposed controller provides a significant reduction in the disturbance influence on the quadrotor tracking performances. Such controller capability is important in real applications where the influence of wind cannot be ignored. Also, low computational cost of the proposed control law, in comparison with similar approaches reported in literature, provides easy implementation in low-cost microcontrollers. The main drawback of the proposed controller is that there are no exact tuning rules for optimal adjustment of filter gains.

Future work will be oriented towards an exact Lyapunov-based stability analysis of the proposed controller with included sliding-mode filters. Such an analysis should provide explicit stability conditions for the controller and filter gains depending on the dynamic model parameters and amplitudes of external disturbances.

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