EVALUATION OF RESISTANCE OF PLANING HULLS IN SMOOTH WATER

Summary

The aim of this paper is evaluation of the total resistance of planing forms with prismatic hull. Computer code has been developed to estimate the total resistance in the planing and preplaning range, which allows the designer to predict the total resistance in the preliminary design phase. Savitsky method is used to evaluate the total resistance in the preplaning and planing speed range, and Savitsky-Brown method for preplaning speed range. Code is tested on Series 62 models because of available experimental data from towing tank tests conducted at the Brodarski Institute in Zagreb. The total resistance obtained by the program code is compared with the measurements of the total resistance. From the obtained results it is evident that the Savitsky method is not suitable for resistance evaluation in the range of the volume Froude number $1 \leq Fn_v \leq 2$ because the deviations in results may be up to 45%, while for the higher Froude number range $Fn_v > 2$ gives satisfactory agreement with the measured values with a maximum deviation of up to 20%. For the low values of the volume Froude number the application of Savitsky-Brown method is recommended, which gives deviations of up to 10%.

Key words: resistance, planing, preplaning

PROCJENA OTPORA GLISERA U MIRNOJ VODI

Sažetak

Cilj ovog rada je procjena ukupnog otpora glisera s prizmatičnom formom trupa. Razvijen je programski kod za procjenu ukupnog otpora ova dva režima plovidbe koji omogućava projektantu brzu prognozu ukupnog otpora u fazi pretprojekta. Korištena je metoda Savitsky za procjenu otpora za predglisirajuće i glisirajuće područje i metoda Savitsky-Brown za predglisirajuće područje plovidbe. Kod je testiran na modelima Serije 62 za koju postoje mjerenja provedena u bazenu Brodarskog instituta u Zagrebu. Ukupni otpor dobiven programskim kodom uspoređen je s rezultatima mjerenja ukupnog otpora. Iz dobivenih rezultata vidljivo je da metoda Savitsky nije pogodna za proračun otpora u području niskih vrijednosti Froudeovog broja na temelju istisnine $1 \leq Fn_v \leq 2$, jer odstupanje u rezultatima može iznositi do 45%, dok za područje većih vrijednosti Froudeovog broja $Fn_v > 2$ daje zadovoljavajuće slaganje s izmjerениm vrijednostima uz maksimalno odstupanje do 20%. Za područje niskih vrijednosti Froudeovog broja preporuča se primjena metode Savitsky-Brown kod koje su odstupanja do 10%.

Ključne riječi: otpor, glisiranje, predglisiranje
1. Introduction

We are witnessing an unexpected large increase in interest in fast craft, not only in traditional but also in new areas of application. One of the prerequisites of a successful project of a fast craft is its appropriate hydrodynamic properties. Predicting the resistance of a planing hull has been of interest to naval architects for several decades. Even though considerable amount of research has been carried out in this area, there remains a degree of uncertainty in the accurate resistance prediction in the early design stage.

Two different pressure types are acting on the planing hull: hydrostatic pressure and hydrodynamic pressure. Hydrostatic pressure is known as buoyancy which is proportional to displacement of the vessel. Hydrodynamic pressure is dependent on the flow around hull and proportional to the velocity square. In general, when the Froude number is less than 0.4, hydrostatic forces (buoyancy) are more predominant than the hydrodynamic pressure forces. Vessels in this speed zone are called displacement vessels. When Froude number is between 0.4 and 1.0 (likewise 0.5 and 1.2), vessels in this speed zone are called semi-displacement vessels. Finally, when the Froude number is greater than 1.0 (likewise 1.2) hydrodynamic forces have an impact on hull and create lift; these vessels (in this speed region) are called planing hulls. Generally, the planing range starts for volume Froude number $F_{n_v} > 1.2$ and $F_{n_v} = 1.0$ being the lower limit for the planing range [1]. For this reason, it is possible to categorize high speed vessels with respect to the volume Froude number, as well as hull forms and their resistances. Fundamental speed ranges are preplaning (up to approximately $F_{n_v}$ 2.5), semi-planing (approximately from $F_{n_v}$ 2.5 to 4.0) and fully planing (approximately from $F_{n_v}$ 4.0 and above).

Resistance prediction methods generally fall into the following categories: planing hull series, prismatic equations, numerical methods, empirical calculations and theoretical methods. All prediction methods mentioned above are based on the same data, experiments, and observations of planing hull models tests. Prismatic bodies have constant cross section and straight buttocks through length. Most of planing hulls can be examined as a prismatic because during planing stage, the sections of hull underwater are constant. There are three prismatic resistance prediction methods: Savitsky, Shuford/Brown and Lyubomirov method. The resistance difference between these methods is usually less than 10%. Savitsky method gives the highest prediction and the other two give a lower prediction [2].

Daniel Savitsky [3] published in 1964 comprehensive paper which summarized previous experimental studies on the hydrodynamics of prismatic planing surfaces and presented a method for application of these results to design. Originally, the reason for exploring the planing phenomena was a waterbased aircraft, and in the later years of the relevant period, the emphasis was placed on the planing boats and hydrofoil craft. The paper written by Mercier and Savitsky [4] deals with the calculation of the total resistance of transom-stern craft in the preplaning range, specifically for volume Froude numbers between 1 and 2. The predictive technique is established by regression analysis of the resistance data of seven transom-stern hull series (NPL, Nordstorm, De Groot, SSPA, Series 64, Series 63 and Series 62). It should be mentioned that of all systematic series mentioned only Series 62 is hard chine form.

The main purpose of this paper is to evaluate the total resistance of Series 62 models and compare them with the experimental results. Computer code has been developed to estimate the total resistance in the planing and preplaning range, which allows the designer to predict the total resistance in the preliminary design phase.
2. Hydrodynamic phenomena related to planing hulls in smooth water

Hydrodynamic phenomena associated with transom-stern hulls when running in smooth water over a wide speed range will be described briefly. At zero and low speed, planing crafts are displacement hulls, obtaining their entire lift by buoyant forces. Furthermore, as speed increases to a speed coefficient, \( C_r = \sqrt{\frac{g}{B_r}} \approx 0.5 \), there appears the first visual evidence of the influence of dynamic effects upon the flow patterns. Complete ventilation of the transom occurs and appears to be independent of deadrise, trim, or hull length for typical values of these parameters. At speed coefficients between 0.5 and 1.5, the dynamic effects produce a positive contribution to lift, although, in most cases, not sufficient to result in a significant rise of the center of gravity or emergence of the bow. Generally, the flow has only slightly separated from the forward length of the chine so that there is significant side wetting. In this speed range, the craft is essentially a high-speed displacement hull. At speed coefficient larger than approximately 1.5, a well-designed planing boat should develop dynamic lift forces which will result in a significant rise of center of gravity, positive trim, emergence of the bow, and separation of the flow from the hard chines [4].

It has been found that the flow which separated from the chine may reattach to the side of the prismatic hull at some distance forward of the transom for certain combination of speed coefficient \( C_r \), angle of deadrise \( \beta \), trim angle \( \tau \), and mean wetted length-beam ratio \( \lambda \). An empirical formulation and confirming test data for defining the extent of side wetting are given in [4]:

\[
\lambda_{v1} - \lambda_{v2} = 3C_r^2 \sin \tau
\]  

(1)

To define the operating conditions for the chines-dry case, \( \lambda_{v2} \) must be equal to zero. From the relations given in [3], it can be easily shown that:

\[
\lambda_{v1} \equiv \lambda - \frac{1}{2 \pi} \frac{\tan \beta}{\tan \pi} \lambda
\]  

(2)

where

\[
\lambda = \frac{L_k + L_c}{2b}
\]  

(3)

Wetted keel length is given by following expression:

\[
L_k = \frac{d}{\sin \tau}
\]  

(4)

and \( L_c \) is wetted chine length, Fig.1.

Thus, for chines-dry planing of a prismatic form, following equation has to be satisfied:

\[
C_r^2 \lambda = \frac{0.16 \cdot \tan \beta}{\tan \tau} \frac{\lambda}{3 \sin \tau}
\]  

(5)

If the trim angle is 1.5 degrees, there will not be significant accumulation of water under the hull. If the trim angle is larger than 1.5 degrees, accumulation of water is greater and wetted area is getting towards the spray root [3]. Wagner had made a mathematical study of the flow at the leading edge of a planing surface of infinite length and found that the rising water surface blends into a thin sheet of water flowing forward along the planing surface. Due
to that, the maximum pressure is right behind the point of spray root, which can be seen in Figure 2.

![Fig. 1 Waterline](image)

**Fig. 1 Waterline**

*Slika 1. Linije dodira s vodenom površinom za prizmatičnu plohu*

![Fig. 2 Pressure distribution on flat planing surface](image)

**Fig. 2 Pressure distribution on flat planing surface**

*Slika 2. Raspodjela tlaka na ravnu glisirajuću površinu*

![Fig. 3 Wave rise on flat planing surface](image)

**Fig. 3 Wave rise on flat planing surface**

*Slika 3. Val na ravnu glisirajuću površinu*

Total wetted area at planing hulls can be separated into two areas:

- wetted area behind spray root
- wetted area in front of spray root.

Planing hulls have trim that usually has its maximum value at values of speed coefficient 1.5 to 2.0. If the speed is increasing, trim will decrease, but the wetted area of keel will increase.
2.1. Prismatic hull form

A prismatic hull has a constant cross section and straight buttocks along the hull’s entire length. Many pure-planing hulls can be considered prismatic because the sections of the hull in contact with the water are constant when planing. In the speed range where craft reaches planing range, when the flow has separated from the chines and transom and the wetted keel length is less than LWL, computational methods are available for prediction of hull performance [3]. Predictions include trim, wetted keel and chine lengths, draft and resistance like functions of load, speed, transverse deadrise angle and longitudinal center of gravity. The lift coefficient for a finite-deadrise surface $C_{L\beta}$ is related to that for a flat-bottom surface $C_{L0}$ by the following equation:

$$C_{L\beta} = C_{L0} - 0.0065\beta C_{L0}^{0.6}$$  \(6\)

$$C_{L0} = \frac{\Delta}{0.5\rho v^2 B_{px}^2}$$  \(7\)

where $B_{px}$ is maximum chine beam.

Controllable transom flaps are sometimes used to control high-speed crafts trim with the goal to enhance the performances and characteristics of maneuverability. Also they are used for decreasing the resistance force [4]. If the flaps are used, their lift should be taken into account. The flap lift is given by:

$$\Delta_f = 0.046L_F \rho b \sigma \left[ \frac{D}{2v^2} \right]$$  \(8\)

Since the flap increases the hydrodynamic lift, there is also an increase in the form drag, $\Delta \cdot \tan \tau$, where $\Delta$ is the total lift (including the flap lift) and is equal to the craft weight. Pressure on the flap causes a drag which is proportional to the flap lift:

$$D_F = 0.0052\Delta_f (\tau + \delta)$$  \(9\)

The flap lift acts on the distance of 0.6b ahead of the trailing edge of the flap.

3. Preplaning resistance

Mercier and Savitsky [5] conducted a regression analysis of the smooth water resistance data of seven transom-stern hull series which included 118 separate hull forms. An analytical procedure was developed for predicting the resistance of transom-stern hulls in the preplaning range where volume Froude numbers is less than 2. Brown completed an experimental and theoretical study of planing surfaces with trim flaps.

3.1. Procedure and equations

For preplaning range, four parameters were selected for inclusion in the resistance equation:

$$X = \frac{\sqrt[3]{\nabla}}{L_{WL}}$$  \(10\)

$$Z = \frac{\nabla}{B_{px}^3}$$  \(11\)
All dimensions in Eqs. (10)-(13) should be measured from the lines plan at the stillwater draft and trim. \( B_x \) and \( A_x \) are the maximum waterline beam and transverse section area respectively. Least-squares curve-fitting was applied and following equation for resistance calculation with 27 terms was obtained:

\[
\frac{R_r}{\Delta} = A_1 + A_2 X + A_3 Z + A_4 U + A_5 W + A_6 XZ + A_7 XU + A_8 XW + A_9 ZU + A_{10} ZW + A_{11} UW + A_{12} X^2 + A_{13} Z^2 + A_{14} U^2 + A_{15} W^2 + A_{16} XZ^2 + A_{17} XU^2 + A_{18} XW^2 + A_{19} ZU^2 + A_{20} ZW^2 + A_{21} ZW^2 + A_{22} U^2 + A_{23} UZ^2 + A_{24} UW^2 + A_{25} WX^2 + A_{26} WZ^2 + A_{27} WU^2
\]  

Eliminating the number of terms, which are of minor influence on the final result, in equation (14) is beneficial in two reasons:
1. It is true that with more term, equation could give the better and correct results. But with more unknown terms, dependence of the main parameters (\( X, Z, U \) and \( W \)) would be less.
2. Equation with fewer unknown terms is easy to calculate, it is not necessary to have computational software.

After elimination, the main equation (14) for resistance has 14 unknown terms:

\[
\frac{R_r}{\Delta} = A_1 + A_2 X + A_3 U + A_4 W + A_5 XZ + A_6 XU + A_7 XW + A_8 ZU + A_{10} ZW + A_{11} W^2 + A_{12} XW^2 + A_{13} ZU^2 + A_{14} UW^2 + A_{15} WU^2
\]  

Unknown coefficients \( A \) are given in Table 1. for each volume Froude number in the range \( 1 \leq \bar{Fn}_v \leq 2 \) (the range of for the preplaning condition). These coefficients were calculated upon the towing tank experimental results conducted by Mercier and Savitsky.

Equation (15) applies to craft with 44482 kg displacement. For other values of displacement, the results can be corrected according to the relation:

\[
\frac{R_r}{\Delta}_{\text{corr}} = \left( \frac{R_r}{\Delta} \right)_{\text{orig}} + \left[ (C_F + C_A) - C_{FA} \right] \frac{1}{2} \frac{S}{\sqrt{\frac{1}{3}}} \frac{F_n^2}{\Delta^2}
\]  

where

\[
\frac{0.242}{\sqrt{C_{FA}}} = \log \left( \frac{R_n C_{FA}}{2} \right)
\]  

\[
R_n = \frac{F_n L}{\sqrt{\frac{32.2}{64}}} \sqrt{32.2 \cdot \frac{100000}{1.2817 \cdot 10^{-5}}}
\]  

\[
C_F = \frac{0.075}{(\log R_n - 2)^2}
\]
The wetted area for the models with transom sterns may be estimated from the following equation:

\[
S / \sqrt{V^{2/3}} = 2.262 \sqrt{\frac{L_{WL}}{V^{1/3}}} \left[ 1 + 0.046 \frac{B_X}{T} + 0.00287 \left( \frac{B_X}{T} \right)^2 \right]
\]  

(20)

Alternative equation for wetted surface that depends on the block coefficient is:

\[
S / \sqrt{V^{2/3}} = \left( \frac{L_{WL}}{V^{1/3}} \right)^2 \left[ 1.7 \frac{B_X}{L_{WL}} + \frac{B_X}{L_{WL}} \cdot C_B \right]
\]

(21)

4. Planing resistance

The total hydrodynamic resistance of a planing surface is composed of pressure resistance, acting normal to the wetted surface, and of viscous resistance acting tangential to the bottom in both the pressure area and spray area [3]. If the fluid is non-viscous, tangential component is equal to zero, so the pressure resistance component \( D_p \) is defined as:

\[
D_p = \Delta \tan \tau
\]  

(22)

If we add to equation (22) viscous resistance component \( D_f \), the total resistance will be equal to:

\[
D = \Delta \tan \tau + \frac{D_f}{\cos \tau}
\]  

(23)

where

\[
D_f = \frac{C_{FA} \rho v_i^2 A b^2}{2 \cos \beta}
\]

(24)

where \( v_i \) is average bottom velocity which is less than the forward planing velocity \( v \) because the planing bottom pressure is larger than the free stream pressure.

In a case of a zero deadrise hull, the dynamic contribution to planing lift is:
\[ C_{ld} = 0.0120 \lambda^{1/2} \tau^{1.1} \] (25)

The dynamic load on the bottom is defined as:
\[ \Delta_d = \frac{1}{2} \rho v^2 b^2 C_{ld} \] (26)

The average dynamic pressure is:
\[ p_d = \frac{\Delta_d}{\lambda b^2 \cos \tau} = \frac{1}{2} \rho v^2 c^2 \frac{(0.0120 \lambda^{1/2} \tau^{1.1})}{\lambda b^2 \cos \tau} = \frac{0.0120 \rho v^2 \tau^{1.1}}{2 \lambda^{1/2} \cos \tau} \] (27)

By applying Bernoulli’s equation between free stream and the bottom of planing surface:
\[ v_1 = \sqrt{v \left( 1 - \frac{2p_d}{\rho \lambda v^2} \right)} \] (28)

Substituting equation (27) into equation (28), one obtains the following expressions for \( \beta = 0 \):
\[ v_1 = \sqrt{\frac{1 - \frac{0.0120 \tau^{1.1}}{\lambda^{1/2} \cos \tau}}{2 \lambda^{1/2} \cos \tau}} \] (29)

If the deadrise angle is different from zero, following equations apply:
\[ C_{l,\beta} = C_{l,0} - 0.0065 \beta C_{l,0}^{0.6} \] (30)

Obviously \( C_{l,0} = C_{ld} \) if \( \beta \neq 0 \).

Substituting equation (25) into equation (30) and (26) and (30) into (27) respectively one obtains [3]:
\[ C_{l,\beta} = (0.0120 \lambda^{1/2} \tau^{1.1}) - 0.0065 \beta (0.0120 \lambda^{1/2} \tau^{1.1})^{0.6} \] (31)
\[ p_d = \frac{\rho v^2 \left[ (0.0120 \lambda^{1/2} \tau^{1.1}) - 0.0065 \beta (0.0120 \lambda^{1/2} \tau^{1.1})^{0.6} \right]}{2 \lambda \cos \tau} \] (32)

Substituting equation (32) into equation (29) it follows:
\[ \frac{v_1}{v} = \left( 1 - \frac{\left[ (0.0120 \lambda^{1/2} \tau^{1.1}) - 0.0065 \beta (0.0120 \lambda^{1/2} \tau^{1.1})^{0.6} \right]}{\lambda \cos \tau} \right)^{1/2} \] (33)

Total resistance of a planing surface is given by following expression:
\[ D = \Delta \tan \tau + \frac{D_f}{\cos \tau} = \Delta \tan \tau + \frac{2 \cos \beta}{\cos \tau} = \Delta \tan \tau + \frac{C_f \rho v^2 (\lambda b^2)}{2 \cos \beta \cos \tau} \] (34)
It must be mentioned that equation (34) can be applied for models whose trim angle does not exceed the angle of four degrees. If the trim angle has higher value, spray root area must be calculated as well, because than it has great influence on total resistance value.

5. Comparison of results

In order to gain insight into the accuracy of methods for the resistance prediction, the results obtained by the methods were compared with the measured values. Series 62 model tests were conducted in the towing tank at the Brodarski Institute in Zagreb [6].

Series 62 models have single chine along the whole hull which separates the bottom from the side; the deadrise angle at the stern part is constant and has value of 12.5 degrees which enables good planing characteristics; and bow frames are convex. An example of the body plan for the series is shown in Fig. 4.

Models M4665 and M4668 were selected from an extensive database of experimental data and their main particulars are given in Table 2. The results for the other models are available in the literature [7], [8]. For each of the models were chosen four tests and input data for preplaning resistance calculation are given in Table 3. Comparison of measured and calculated total resistance as a function of the volume Froude number for models M4665 and M4668 are shown in Figs. 5 and 6.

![Fig. 4 Series 62 body plan](image)

**Table 2 Main particulars of models**

<table>
<thead>
<tr>
<th>Particulars \ Model</th>
<th>M4665</th>
<th>M4668</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_p \ [m^2]$</td>
<td>0.601</td>
<td>0.884</td>
</tr>
<tr>
<td>$L_p \ [m]$</td>
<td>1.192</td>
<td>2.438</td>
</tr>
<tr>
<td>$B_{xx} \ [m]$</td>
<td>0.504</td>
<td>0.363</td>
</tr>
<tr>
<td>$B_{xx} \ [m]$</td>
<td>0.596</td>
<td>0.443</td>
</tr>
<tr>
<td>$B_{TT} \ [m]$</td>
<td>0.477</td>
<td>0.285</td>
</tr>
<tr>
<td>$L_0/B_{xx}$</td>
<td>2.37</td>
<td>6.72</td>
</tr>
<tr>
<td>$L_0/B_{xx}$</td>
<td>2.00</td>
<td>5.50</td>
</tr>
<tr>
<td>$P_{xx}/B_{xx}$</td>
<td>1.18</td>
<td>1.22</td>
</tr>
<tr>
<td>$B_{TT}/B_{xx}$</td>
<td>0.80</td>
<td>0.64</td>
</tr>
<tr>
<td>Centroid of $A_p \ [% \ LP \ fwd \ of \ transom]$</td>
<td>47.5</td>
<td>48.8</td>
</tr>
<tr>
<td>Angle of a-b chine in plan view \ [deg]</td>
<td>5.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Half-angle of waterline entrance \ [deg]</td>
<td>58</td>
<td>39</td>
</tr>
</tbody>
</table>
Table 3 Input data for preplaning resistance calculation for model 4665 and 4668

<table>
<thead>
<tr>
<th></th>
<th>model 4665</th>
<th>model 4668</th>
</tr>
</thead>
<tbody>
<tr>
<td>test 3</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>test 4</td>
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<td>1.10</td>
</tr>
<tr>
<td>test 5</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>test 6</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>test 7</td>
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<td>2.25</td>
</tr>
<tr>
<td>test 8</td>
<td>2.25</td>
<td>2.25</td>
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<tr>
<td>test 9</td>
<td></td>
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</tr>
</tbody>
</table>

6. Conclusion

An empirically based resistance equation may be used to estimate the resistance of craft whose characteristics fall within the range of characteristics embodied in the models whose resistance data were applied to derive the equation. Attempts to estimate resistance of craft which do not have such characteristics must be considered speculative to a greater or lesser extent [4].

Comparing the values of the predicted total resistance in the planing range obtained by Savitsky method with resistance values obtained by the tests in the towing tank, it is evident that for the volume Froude number $F_{n_v} > 2$ there is a satisfactory agreement with the experimental values. The difference between measured and calculated values for all models is up to 20% [8]. Savitsky method is not recommended for calculating the resistance for small volume Froude number values, since the difference between measured and calculated values is up to 45%. For the range of the volume Froude number $1 \leq F_{n_v} \leq 2$ it is recommended to apply Savitsky-Brown method for which the difference between measured and calculated values is less than 10% for all models [8]. The method is in reasonable agreement with the test data.

Preplaning range ends at $F_{n_v} = 2$ and the total resistance calculated by Savitsky-Brown method significantly overpredicts the measured value and the trend of the line does not follow the trend of the line of experimental data.

The prismatic model equations do not include the air resistance which makes a small contribution; however, it should not be neglected. The airflow can also influence trim and sinkage, which again affect the resistance.
Figure 5. Comparison of measured and calculated total resistance for models 4665 and 4668

Slika 5. Usporedba izmjerjenog i izračunatog ukupnog otpora za modele 4665 i 4668
**Figure 6.** Comparison of measured and calculated total resistance for models 4665 and 4668 $1 \leq F_{n_v} \leq 2$

**Slika 6.** Usporedba izmjerenog i izračunatog ukupnog otpora za modele 4665 i 4668 za $1 \leq F_{n_v} \leq 2$
REFERENCES