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STRUCTURAL DESIGN OF A TYPHOON CLASS SUBMARINE

Abstract

Submarine structural design, based on the theory of shells of revolution, is one of the demanding tasks and key factors that ensure the submarine operability, especially in the case of Typhoon submarine class being the largest submarine class in the world. In the submarine structural design it is essential to ensure, addition to structural integrity, the maximum possible submarine deadweight in order to enable the necessary equipment and armament setting. The Typhoon class submarine structure is presented in this paper along with its design using the graphical optimization method, with the largest deadweight as the design objective while satisfying the different hull strength criteria.

Key words: submarine, Typhoon class, structural design, theory of shells of revolution

PROJEKTIRANJE KONSTRUKCIJE PODMORNICE KLASE TYPHOON

Sažetak

Projektiranje trupa podmornice, kao jednog od najbitnijih čimbenika koji omogućuju i osiguravaju njezinu operativnost, je zahtjevan zadatak temeljen na teoriji čvrstoće ljusaka, osobito u slučaju klase Typhoon, najveće klase podmornica na svijetu. Prilikom projektiranja trupa podmornice potrebno je, osim kriterijima čvrstoće, posebnu pažnju posvetiti smanjenju mase trupa kako bi se što više povećala nosivost podmornice radi smještaja opreme i naoružanja. Ovim je radom prikazana konstrukcija podmornice klase Typhoon te je izvršena optimizacija grafičkom metodom s ciljem povećanja nosivosti podmornice i istovremenog zadovoljenja različitih kriterija čvrstoće trupa.

Ključne riječi: podmornica, Typhoon klasa, projektiranje konstukcije, teorija rotacijskih ljuski

1. Introduction

Submarine is a ship capable of sailing on or under the water surface and of independent emerging or submerging at any moment. The term "submarine" most commonly refers to large, crewed and autonomous vessels and differs from the term "submersible", which refers to vessels of more limited underwater capability [1]. Today, submarines are mainly used in military purposes and, to a lesser extent, for scientific-research and for engineering or tourist purposes.

The worldwide military application and development of submarines started at the beginning of 20th century when the appearance of almost undetectable underwater ships intrigued the world superpowers. During the Second World War, the Japanese were building, the biggest submarines of the time – the "Sen Toku" class [2]. The Sen Toku submarine class was unique, not only by its enormous length of 122 m but also by its structure. It was the world's first "catamaran" type submarine. The internal structure contained two pressure hulls placed parallel to each other with the third pressure hull placed on them as a hangar for three smaller folding planes. The Sen Toku class was technologically important since the design of the big watertight hangar led to the construction of the USA submarine ballistic missile silo and enabled the Typhoon submarine class design.

2. Typhoon class submarine structure

The Typhoon submarine class is the world's largest submarine class carrying 20 R-39 rockets, Figure 1, with a striking range of 8300 km. Each rocket is armed with 10 nuclear warheads and it has a total mass of 84 tons [3]. The total number of six Typhoon type submarines has been built. Each submarine has the length of about 175 m, width of 23.3 m, height 25 m and the displacement of 48000 tons, [4], which makes them the world's largest submarines. The unique characteristic of Typhoon class is its structure containing 5 pressure hulls. Two of them are parallel to each other and they are stretching from the stern to the bow (hereinafter referred to as „main hulls") and 3 smaller ones placed above them in the center line on the stern, in the middle and in the bow part of submarine, Figure 2.

Fig. 1. Typhoon class "clay "model [5]

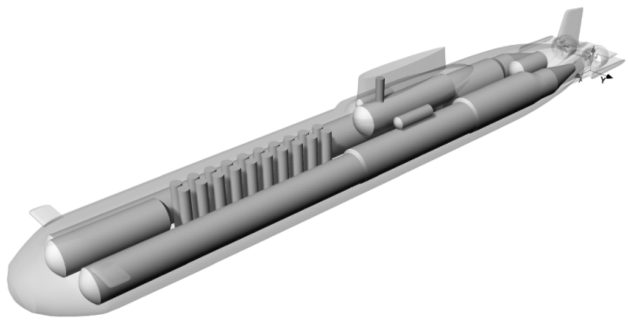

Fig. 2. Layout of pressure hulls

Main hull diameters, having one common guideline in spite of being concentric to each other, are different for particular submarine parts, Figure 3. The bow diameter of the main hulls is 7.2 m, [7],

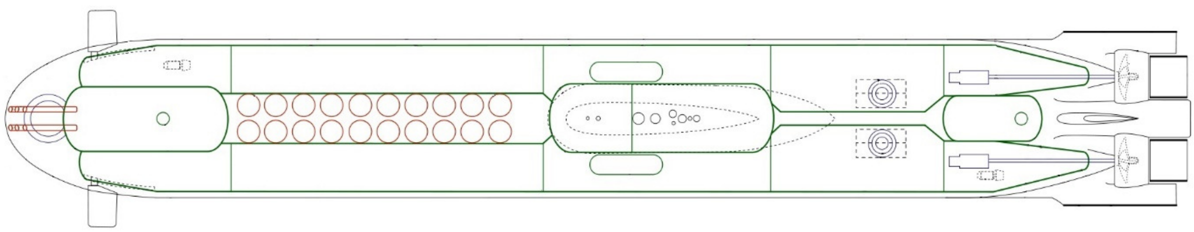

Fig. 3. Layout plan of pressure hulls arrangement [6]

the distance between the light hull and main hulls is 1.2 m and the R-39 rocket diameters are 2.4 m [3]. Since the submarine breadth is 23.3 m it is simple to determine the stiffening ring heights (height of web and flange's thickness) should not exceed 45 cm at the bow part of the main hulls.

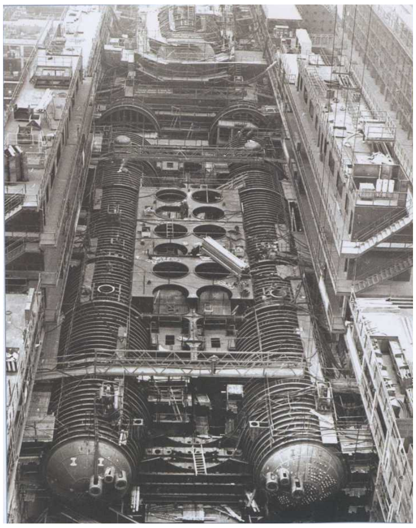

Fig. 4. Archival footage during the submarine construction [8]

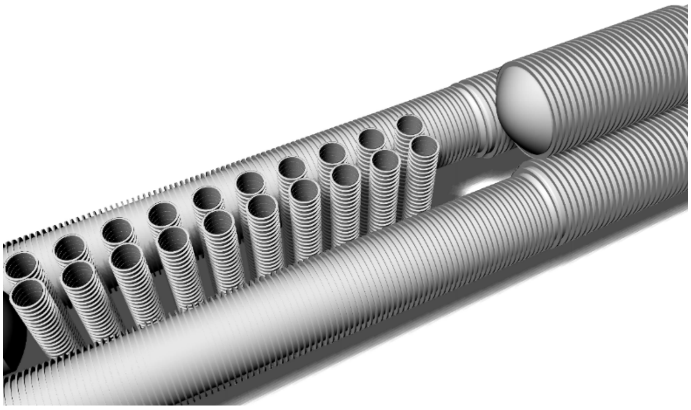

Fig. 5. Pressure hulls with stiffening rings

3. Structural design

Structural design is an important issue related to development of any type of engineering structure, particularly ship and submarine structure. It offers a possibility to design such structure that satisfies both its purposes and different structural constraints in an optimal way. For the purposed of the design procedure it is inevitable to define design objective, variables and design constraints. In some cases the design procedure can be very complex and an application of various computer programs is required in order to deal with them successfully in reasonable time. Optimization methods such as linear programming, genetic algorithms, differential evolution etc. are often applied. In some simpler cases, as the present one, graphical method, being the simplest one, is suitable enough.

4. Typhoon class submarine structural design

The Typhoon class pressure hulls are made of titanium, [9]. For the purpose of calculation, the Ti-6Al-4V (Grade 5) titanium alloy will be used, with main properties given in Table 1.

Table 5. The properties of titanium alloy [10]

Density, ρ	4430 kg/m^3
Tensile Yield Strength, σ_t	880 MPa
Modulus of Elasticity, E_{st}	113.8 GPa
Poison's Ratio, ν	0.342

Main data related to the dive depth, [11], required for calculation are defined as: operational depth, $H_0 = 350$ m, limiting operational depth, $H_l = 380$ m, diving depth limit, $H_d = 500$ m, safety of decline, $\Delta H = 20$ m. Other constraints required for calculation are defined as: sea density, $\rho = 1025.87\ \mathrm{kg/m^3}$, gravity constant, $g = 9.806650\ \mathrm{m/s^2}$, radius of observed cylinder, $r_C = 3.6$ m; titan density, $\rho_{ti} = 4430\ \mathrm{kg/m^3}$, $\sigma_t = 8.80 \cdot 10^8$ Pa, modulus of elasticity, $E_{st} = 1.138 \cdot 10^{11}$ Pa, Poison's ratio, $\nu = 0.342$.

The whole calculation is based on the procedure in [12] and [13]. Based on the defined properties, the safety factor, K_S and the design pressure, p, can be determined as

$$K_s = \frac{H_d}{H_l} = 1.316 \,, \tag{1}$$

$$p = K_s \cdot \rho \cdot g \cdot (H_L + \Delta H) = 5.29 \cdot 10^6 \; Pa \,. \tag{2}$$

The main variables are: spacing between stiffening rings, *l*, stiffening ring surface, *A*, plating thickness, *h*. Constraints are: the height of stiffening rings limited to 45 cm and the objective of the lowest structure mass.

4.1.Stiffening ring geometrical properties

Main stiffening ring geometrical properties are defined as flange beam, *a*, flange thickness, h_f, web height, *b*, web thickness, h_s, plating thickness, *h,* and spacing between stiffening rings, *l,* Figure 8. Due to simplicity, the variables *a*, h_f, *b* and h_s are replaced with the stiffening ring cross-section area, *A*.

4.2. Objective function

As specified above, minimization of submarine mass is defined as a design objective. The lowest hull mass can be determined through the objective function, f_G, determined as a ratio between the hull mass, *G*, and the mass of displaced water, *U*.

$$f_G = \frac{G}{U} \,, \tag{3}$$

where *G* and U are defined as

$$G = 2 \cdot r \cdot \pi \cdot \rho_{st} \cdot (A + l \cdot h) \tag{4}$$

$$U = r^2 \cdot \pi \cdot l \cdot \rho \tag{5}$$

Inserting (4) and (5) into (3) yields the final objective function f_G :

$$f_G = \frac{G}{U} = \frac{2 \cdot r \cdot \pi \cdot \rho_{st} \cdot (A + l \cdot h)}{r^2 \cdot \pi \cdot l \cdot \rho} \,. \tag{6}$$

From equation (6) it can be seen that the only unknown variables are the height of stiffening rings, *h,* cross-section area, *A*.

4.3. Constraint functions

For the purpose of optimization functions bounding the design space have to be defined. Constraint functions are defined for meridional stress, circular stress, stress in the stiffening ring, stiffening ring buckling and weighting function.

Design space domain is created when all these functions are defined and graphically presented in one graph. With weighting factor, the optimal solution is picked from the free design domain space.

4.3.1. Meridional stress constraint function

Meridional stress constraint function, A_1, is defined as:

$$A_1(h) = \frac{l \cdot h}{\lambda_1(h)} \,, \tag{7}$$

where the meridional stress lambda function, λ_1, geometrical shell factor, $u(h)$, function, $N_1(h)$ and *F* functions of geometrical properties, $F_3(h), F_4(h)$, are defined as

$$\lambda_1(h) = u(h) \cdot \left(\frac{1.543 \cdot F_4(h)}{N_1(h)} - F_3(h) \right) , \tag{8}$$

$$u(h) = \left(\frac{\sqrt[4]{3(1-\nu^2)}}{2} \cdot \frac{1}{\sqrt{r_c \cdot h}}\right), \tag{9}$$

$$N_1(h) = \frac{\sigma_t}{p} \cdot \frac{h}{r} - 0.5\,, \tag{10}$$

$$F_3(h) = \frac{\sinh(2\cdot u(h)) + \sin(2\cdot u(h))}{\cosh(2\cdot u(h)) - \cos(2\cdot u(h))}\,, \tag{11}$$

$$F_4(h) = \frac{\sinh(2\cdot u(h)) - \sin(2\cdot u(h))}{\cosh(2\cdot u(h)) - \cos(2\cdot u(h))}\,. \tag{12}$$

4.3.2. Circular stress constraint function

Circular stress constraint function for plastic buckling, $A_{21}(h)$, and circular stress constraint function for elasto-plastic buckling, $A_{22}(h)$,are defined as

$$A_{21}(h) = \frac{l\cdot h}{\lambda_{21}(h)}, \tag{13}$$

$$A_{22}(h) = \frac{l\cdot h}{\lambda_{22}(h)}\,, \tag{14}$$

where the circular stress lambda function for plastic buckling, λ_{21}, function $N_{21}(h)$, geometrical properties functions, $F_1(h)$, circular stress lambda function for elasto-plastic buckling, λ_{22},and function $N_{22}(h)$ are defined as

$$\lambda_{21}(h) = u(h) \cdot \left(\frac{0.85\cdot F_1(h)}{N_{21}(h)} - F_3(h)\right), \tag{15}$$

$$N_{21}(h) = 1 - 0.8 \cdot \left(\frac{\sigma_t}{p} \cdot \frac{h}{r}\right), \tag{16}$$

$$F_1(h) = 2 \cdot \frac{\cosh(u(h))\sin(u(h)) + \sinh(u(h))\cos(u(h))}{\cosh(2\cdot u(h)) - \cos(2\cdot u(h))}\,, \tag{17}$$

$$\lambda_{22}(h) = u(h) \cdot \left(\frac{1.543\cdot F_4(h)}{N_1(h)} - F_3(h)\right), \tag{18}$$

$$N_{22}(h) = 1 - \frac{0.36\cdot\sigma_t}{\frac{p\cdot r}{h} - E_{st}\cdot\frac{h}{r}\cdot\frac{0.12}{u(h)-0.367}}\,. \tag{19}$$

4.3.3. Stiffening ring stress constraint function

Stiffening ring stress constraint function for plastic instability, $A_{31}(h)$, and the stiffening ring stress constraint function for general instability, $A_{32}(h)$, are given as

$$A_{31}(h) = \frac{l\cdot h}{\lambda_{31}(h)}\,, \tag{20}$$

$$A_{32}(h) = \frac{l\cdot h}{\lambda_{32}(h)}\,, \tag{21}$$

where the stiffening ring lambda stress constraint function for plastic instability $\lambda_{31}(h)$, function, $N_{31}(h)$, and the stiffening ring lambda stress constraint function for general instability $\lambda_{32}(h)$, are defined as

$$\lambda_{31}(h) = \frac{u(h)\cdot F_3(h)}{N_{31}(h)}\,, \tag{22}$$

$$N_{31}(h) = \frac{1.7\cdot p_d\cdot r}{h\cdot\sigma_t} - 1\,, \tag{23}$$

$$\lambda_{32}(h) = \frac{-B_0(h)+\sqrt{B_0^2(h)+4B_0(h)\cdot C_0(h)}}{2\cdot A_0(h)}\ , \tag{24}$$

Functions C_0, B_0 and A_0 are given as

$$C_0(h) = b^2 \cdot h \cdot u(h) \cdot F_3(h)\ , \tag{25}$$

$$B_0(h) = \frac{8\cdot p_d \cdot r^3}{E_{st}} \cdot u(h) \cdot F_3(h) - b^2 \cdot h \cdot (4 + 2 \cdot k \cdot u(h) \cdot F_3(h))\ , \tag{26}$$

$$A_0(h) = \frac{8\cdot p_d \cdot r^3}{E_{st}} - b^2 \cdot k \cdot h \cdot (8 - 3 \cdot k \cdot u(h) \cdot F_3(h))\ , \tag{27}$$

and the coefficient k is determined as the ratio between the flange beam and the stiffening ring spacing

$$k = \frac{a}{l}\ . \tag{28}$$

Coefficient k is a parameter which provides the necessary space for placing the equipment between stiffening rings, in the case when stiffening rings are placed on the internal cylinder side. In that case, optimal k is usually determined as 0.18, [11]. Since Typhoon class stiffening rings are placed on the outer side of the cylinder, there is no need for the submarine outfit space. Therefore, coefficient k can be freely chosen between 0 and 1, having in mind, that with a higher k, the stiffening ring cross-section moment of inertia is higher. The chosen k is 0.28.

4.3.4. Stiffening ring buckling constraint function

Constrained function for stiffening ring buckling, $A_4(h)$, is defined as

$$A_4(h) = 0.06 \cdot h_s^2 + a \cdot l \cdot h\ , \tag{29}$$

4.3.5. Weighting function constraint

The weighting function is defined as

$$A_G(h) = \frac{1}{2} \cdot \frac{\rho}{\rho_{ST}} \cdot l \cdot (r_C \cdot f_G - h)\ , \tag{30}$$

where f_g is given by (3). Since the unknown variables in f_g are h and A, they must be chosen from the design space and then the weighting function can be calculated.

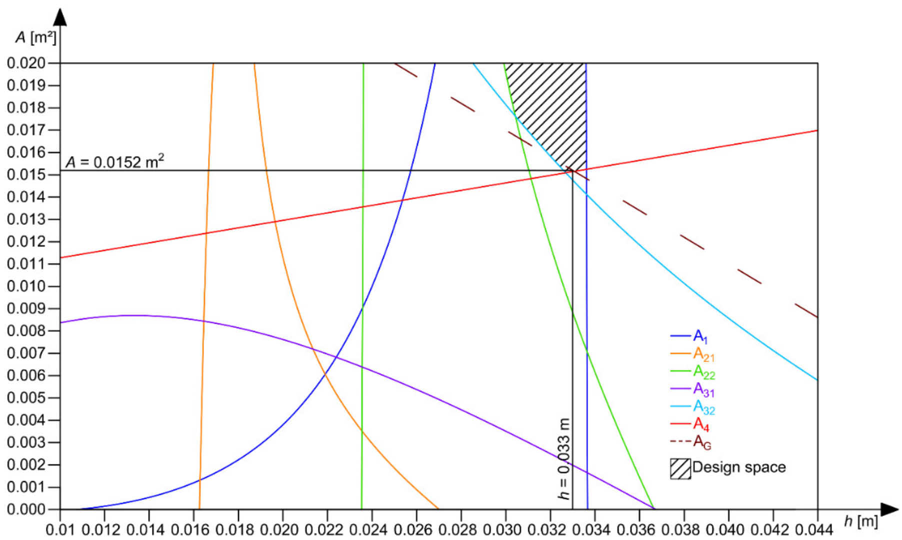

Fig. 6. Free domain space for submarine hull made of titanium with stiffening ring space of 0.6 m

4.3.6. Graphical representation of the design space

The design space, bounded by the constraint functions, for the considered case is presented in Figure 6, with the stiffening ring height, *h*, on the abscissa and the stiffening ring cross-sectional area, *A*, on the ordinate.

From Figure 6, the stiffening ring height and the stiffening ring cross-sectional area, h=0.033 m and A=0.0152 m^2, respectively, are chosen as "better" solutions than h=0.0327 and A=0.0152 m^2 since there are no technological possibilities to manufacture a plate with the tolerance in thickness lower than 0.1 cm. Using the same procedure, solutions for all stiffening ring spacing are determined in Table 3**,**

Table 6. Solutions for varied stiffening ring spacing for the 7.2m diameter cylinder made of titanium

l, m	*h*, m	*A*, m^2	f_G	*a*, m
0.50	0.03	0.0143	0.1406	0.140
0.55	0.032	0.0145	0.1400	0.154
0.60*	0.033*	0.0152*	0.1399*	0.168*
0.65	0.034	0.016	0.1406	0.182
0.70	0.035	0.0165	0.1405	0.196
0.75	0.036	0.0173	0.1417	0.21
0.80	0.037	0.018	0.1427	0.224

*Optimal solution

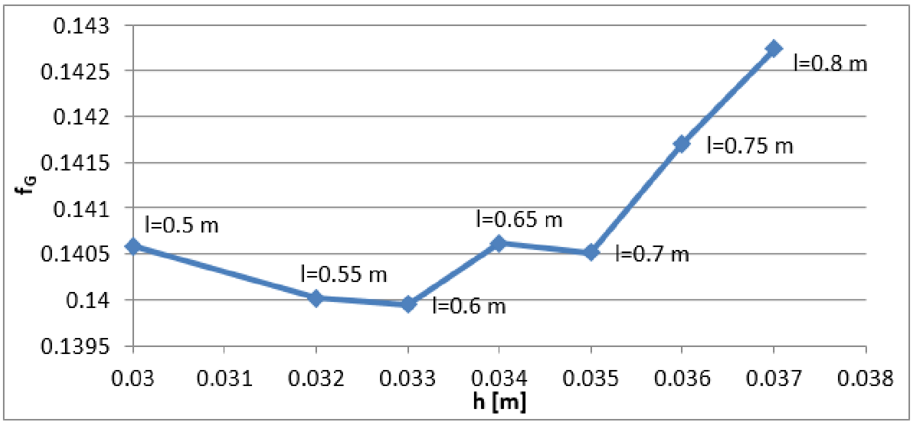

Fig. 7. Relation between the weight function f_G and the plate thickness, *h*, for varied stiffening ring spacing *l*

Objective function, f_G, is presented in Figure 7. It can be noted from Table 3 and Figure 7 that the lowest f_G (0.1399) is determined when *l* = 0.6 m. For that solution, *h* = 0.033 m, *A* = 0.0152 m^2 and *a*=0.168 m.

Based on that the chosen stiffening ring dimensions are the following :

Stiffening ring spacing	l =600 mm
Plate thickness	h =33 mm
Web height	b =400 mm
Web thickness	h_s=25 mm
Flange breadth	a =168 mm
Flange thickness	h_f=33 mm
Stiffening ring area	A =15544 mm^2

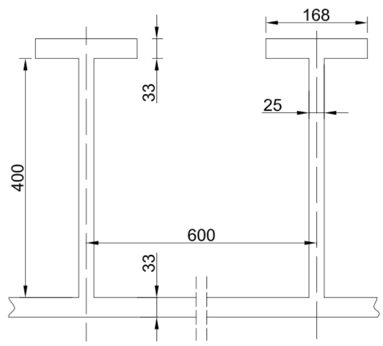

Fig. 7. Stiffening ring geometrical properties

Since the chosen stiffening ring cross-section area is greater than required, it is necessary to calculate a new weight function, f_G= 0.14132. The newly calculated f_G shows that 14% of buoyancy force supports the structure and the rest of it can be used for outfit, armament or as a buoyancy reserve.

5. Hull structure stability

Stiffening ring cross-section moment of inertia is needed for the calculation of hull structure stability. One of the variables needed to determine the moment of inertia is effective length, which is determined by the equation from [14] :

$$L_{ef} = \frac{2}{\sqrt[4]{3(1-\nu^2)}}\sqrt{r \cdot h}\ . \tag{31}$$

Based on Figure 8, the calculated moment of inertia is

$$I = 0.001018862\ \mathrm{m}^4$$

5.1. Plate buckling between stiffening rings

Based on the von Mises criterion the critical stress, p_{cl}, is defined as, [12]

$$p_{cl}[n] = E_{st} \cdot \frac{h}{r} \cdot \frac{1}{n^2+0.5\cdot\alpha^2} \cdot \left(\frac{\alpha^4}{(n^2+\alpha^2)^2} + \frac{1}{12(1-\nu^2)} \cdot \left(\frac{h}{r}\right)^2 \cdot (n^2+\alpha^2)^2\right), \tag{32}$$

where n is the number of half-waves in radial direction, and α is defined as

$$\alpha = \frac{r \cdot \pi}{l} \tag{33}$$

Relation between the critical stress and the number of half-waves is shown in Figure 9.

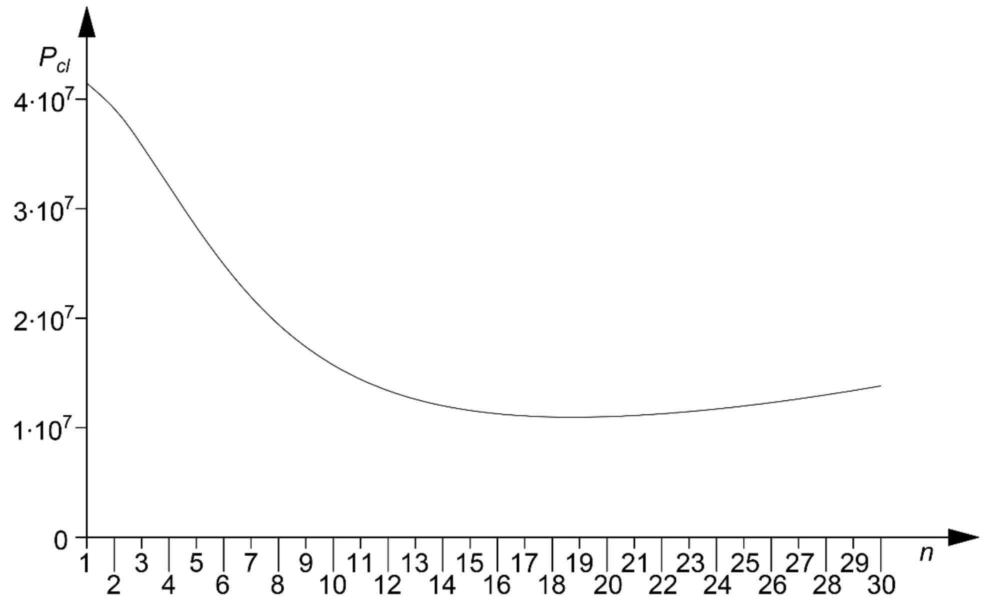

Fig. 9. Relation between critical stress and number of half-waves

Minimal critical stress of $1.1\cdot10^7$ Pa occurs for 19 half-waves. Taking into account 10% for safety, the ratio between critical stress and design pressure has to be

$$\frac{p_{cl}}{p_d} \geq 1.1 \tag{34}$$

Since the proposed solution yields $\frac{p_{cl}}{p_d} = 2.07$, this requirement is fulfilled .

5.2. Stiffening ring plate buckling between bulkheads

The Typhoon class has 19 watertight compartments, [15]. Three of them are in three smaller pressure hulls placed in the centerline and one above the main hulls. Therefore, the main hulls are divided into 16 watertight compartments. Due to a long propeller shaft, rudders and the distance between the main hulls and the light hull, the length of the main hulls can be approximated to 160 m. Based on that assumption, the length of one watertight compartment, L, can be assumed to be 20 meters.

Based on the von Mises criterion [11] the critical stress, p_{cl}, is defined as

$$p_{cL}[m] = \frac{E_{st}}{m^2-1+0.5\cdot\beta^2}\cdot\left(\frac{h}{r}\cdot\frac{\beta^4}{(m^2+\beta^2)^2}+\frac{I_r}{r^3\cdot l}\cdot(m^2-1+\beta^2)^2\right), \quad (35)$$

where m is the number of half-waves in axial direction, L is the distance between bulkheads and

$$\beta = \frac{r\cdot\pi}{L}. \quad (36)$$

Relation between the critical stress and the number of half-waves is shown in Figure 10

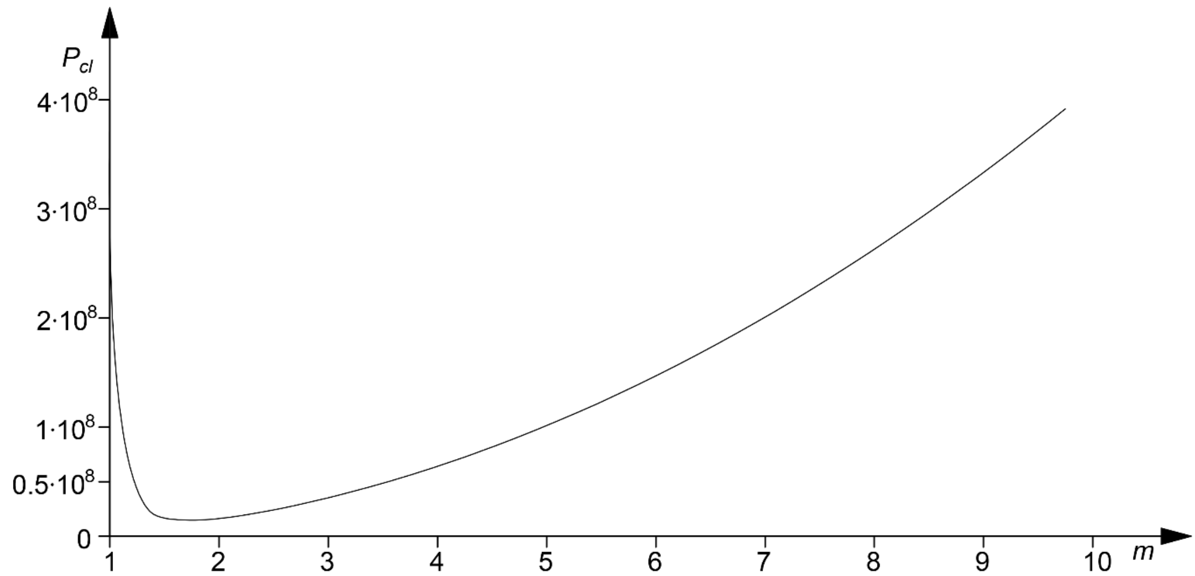

Fig. 10. Relation between the critical stress and the number of half-waves

Minimal critical stress of $1.63\cdot10^7$ Pa occurs for 2 half-waves. Due to material and construction flaws, as well as to a change in the modulus of elasticity in the plastic domain, critical stress is multiplied with η_1 ($\eta_1 = 0.75$) and η_2 ($\eta_2 = 0.768$). Thus, the critical stress is

$$P_{cL} = 9.36\cdot10^6 \text{ Pa}$$

Taking into account the safety of 40%, the ratio between the critical stress and the design pressure has to be

$$\frac{p_{cl}}{p_d} \geq 1.4 \quad (37)$$

Since, in the considered case, this ratio is equal to 1.707, this requirement is fulfilled.

6. Conclusion

Since one submarine of Typhoon class is still in service, all characteristics of Typhoon class complex structure are a military secret. This paper is based on the available data and its goal was to analyze that data using theory of shells of revolution. The main objective was to check the structural integrity of main hulls with the maximum height of stiffening ring. Titanium, as a structural material, fulfils all considered requirements. Also, because of corrosion resistance, lower mass in comparison with HY-80, and non-magnetic properties, titanium is an ideal structural material for submarines. The result is a less detectable, long life submarine with a great buoyancy reserve. Also, these submarines are among the few last submarines in the world made of titanium due to its cost.

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