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PROBABILISTIC MODEL OF ULTIMATE STRENGTH REDUCTION OF GROUNDED SHIP

Abstract

The aim of this paper is to develop probabilistic model of the reduction of the bending moment capacity of oil tanker following grounding accident. The approach is based on Monte Carlo (MC) simulation using probability distributions of damage parameters proposed by International Maritime Organization (IMO). Reduction of ultimate strength is calculated by recently developed regression equations using concept of grounding damage index (*GDI*). Case study is presented for Aframax tanker resulting in the Weibull distribution fitted to the histogram obtained by MC simulation. Results of the study are useful for structural reliability assessment of damaged ships and also for conceptual studies of different structural configurations as demonstrated in the paper.

Key words: damaged tanker, ultimate strength, regression equation

VJEROJATNOSNI MODEL SMANJENJA GRANIČNE ČVRSTOĆE OŠTEĆENOG NASUKANOG BRODA

Sažetak

Cilj ovoga rada je razviti vjerojatnosni model smanjenja graničnog momenta savijanja nasukanog tankera. Pristup se zasniva na Monte Carlo (MC) simulaciji, koristeći raspodjelu vjerojatnosti prema Međunarodnoj pomorskoj organizaciji (IMO). Smanjenje granične čvrstoće računa se regresijskom jednadžbom, koristeći se pojmom pokazatelja oštećenja kod nasukavanja (grounding damage index, *GDI*). U ovom je slučaju analiziran Aframax tanker, za kojeg je histogramu dobivenom MC simulacijom prilagođena Weibull-ova razdioba. Rezultati ovog proračuna mogu biti korisni u procjeni pouzdanosti strukture oštećenog broskog trupa, kao i za pojmovna (konceptualna) istraživanja različitih struktura.

Ključne riječi: oštećeni tanker, granična čvrstoća, regresijska jednadžba

1. Introduction

The structural failure of the tanker may occur due to ship collision, grounding or some other type of human mistake. In case of such an accident, the ship strength could be significantly reduced while still water loads increase and could become considerable cause of the structural overloading. A damaged ship may collapse after a collision or grounding if she does not have adequate longitudinal strength. Such collapse can occur when the hull's maximum load-carrying capacity (or the ultimate hull girder strength, or the bending moment capacity) is insufficient to sustain the corresponding hull girder loads applied. Calculating the ultimate strength after damage is important to determine the options for recovery of the vessel [1-3].

Ship structural designers are unavoidably faced with the question how ship structure would behave in case of the accident. The aim is to avoid breaking of the ship in two parts and sinking of the ship even if the ultimate bending moment capacity is reduced because of the damage. However, from the *a priori* perspective of ship designer, ship damage may occur in a number of ways, while damage parameters are random quantities. Consequently, ultimate longitudinal strength of the damaged vessel is also random value depending on the probability distributions of damage parameters, which are proposed by International Maritime Organization (IMO) [4].

In this paper, probabilistic description of the ultimate longitudinal strength of double-hull oil tanker damaged by grounding is investigated. The assumption proposed by Paik et al. [5] that grounding is caused by conically shaped rock is adopted. Such assumption enables to determine extent of the damage of the inner bottom in cases when height of the damage exceeds height of the double bottom. Reduction of ultimate strength is calculated by recently developed regression equations by Kim et al. [6] using concept of grounding damage index (*GDI*) The procedure begins with draw of random grounding damage parameters by MC simulation: transverse damage location (x_1), damage height (x_2), damage breadth (x_3) and angle of the rock (x_4). Initially, random variables are considered as independent random variables, obeying to the probability distributions proposed by IMO [4]. Rocking angle is calculated based on the drawn height and breadth of the damage. In cases when half of damage breadth ($x_3/2$), added to the transverse damage location (x_1) exceeds half of the ship breadth, damage breadth is reduced to avoid unrealistic damage outside ship's breadth. Once the grounding damage parameters have been defined for each random outcome of MC simulation, the calculation of the grounding damage index (*GDI*) and the residual ultimate longitudinal strength analysis proceed simultaneously. Probability distribution functions are fitted to the histograms representing loss of the intact ultimate bending capacity. Finally, practical applications of results are presented, demonstrating contribution of the study to the state-of-the-art.

2. Calculation methodology

Generally, grounding in double-bottomed structures occurs in both the outer-bottom and the inner-bottom structures. Therefore, the *GDI* should identify the extent and location of grounding damage for both the inner and outer bottom structures, as it is expressed by equation (1). It includes a correction factor (α), see equation 2, to reflect the contribution of the inner bottom structure to the ultimate longitudinal strength of the ship [5].

$$GDI = \frac{A_{ro}}{A_{oo}} + \alpha \frac{A_{ri}}{A_{oi}} \quad (1)$$

where A_{oi} , A_{oo} are original (intact) areas of the inner and outer bottom respectively; A_{ri} , A_{ro} are reduced (damaged) areas of the inner and outer bottom respectively; $A_{ro}/A_{oo} = r_1/B = x_3$; $A_{ri}/A_{oi} = r_2/B$ and r_1 , r_2 are damage breadths in the outer and inner bottom respectively

2.1. Probabilities of damage extent according to IMO

Probability density functions provided by IMO [4] are shown in Figures 1 to 3. Cumulative distribution functions (CDF), calculated by integrating PDFs, are shown in the same figures. They

are adopted as reasonable damage scenarios in terms of non-dimensional transverse damage location (x_1), non-dimensional damage height (x_2) and non-dimensional damage breadth (x_3). Actual damage location, height and breadth are obtained by multiplying x_1 and x_3 by ship breadth B , while x_2 is to be multiplied by ship depth D .

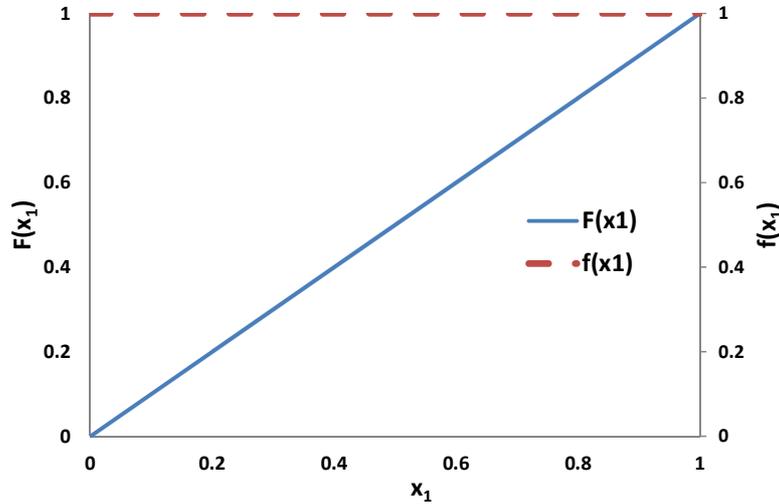


Fig. 1. Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional transverse location of grounding damage (x_1) [4]

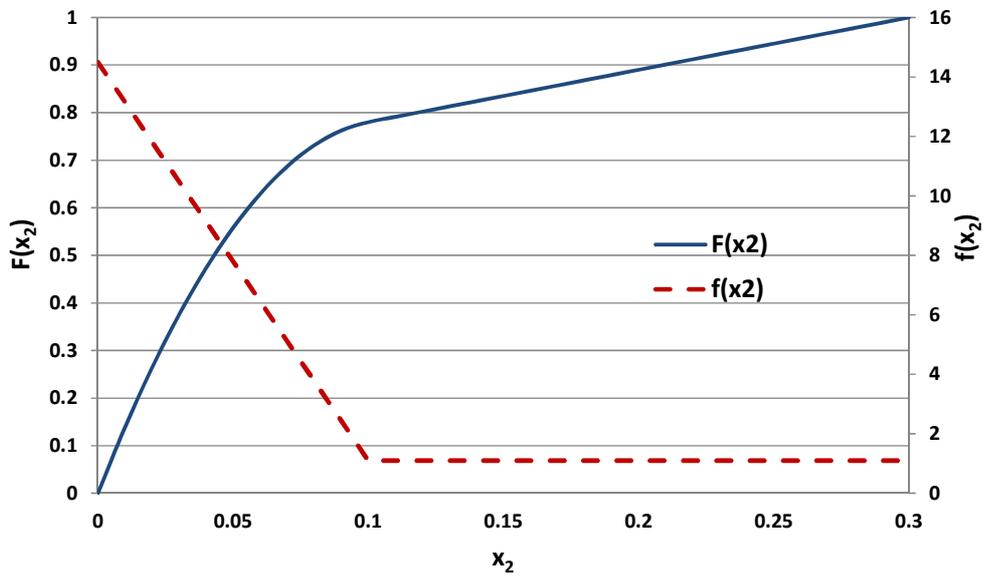


Fig. 2. Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional height of grounding damage (x_2) [4]

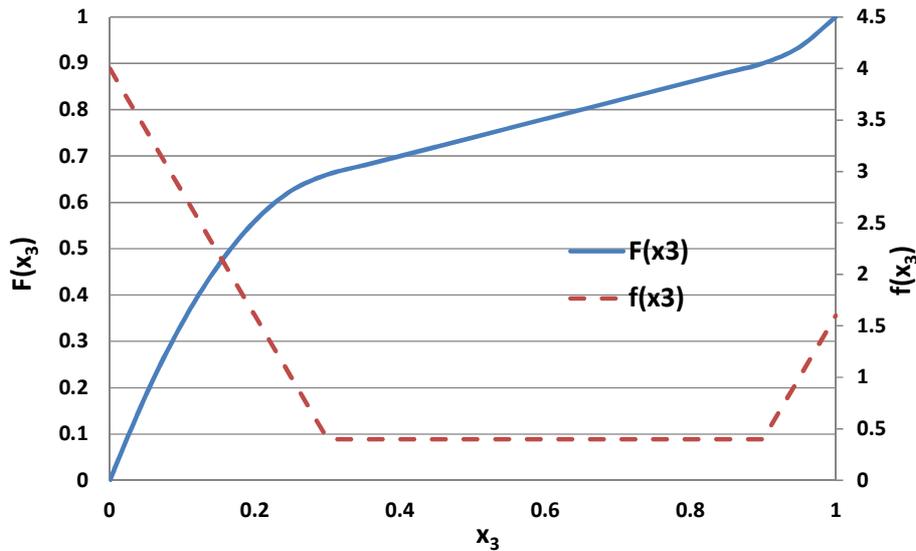


Fig. 3. Probability density function (PDF) and cumulative distribution function (CDF) of the non-dimensional grounding damage breadth (x_3) [4]

2.2. Calculation of reduction of bending moment capacity by Paik’s method

Paik's method procedure includes the following steps: definition of the correction factor α , calculation of the *GDI* and calculation of the ultimate strength reduction by the regression equation.

The correction factor (α) can be determined by the ratio of the slopes (i.e. direction coefficients) of the curves (approximately straight lines) which are representing the influence of inner and outer bottom structures (A_{ri}/A_{oi} and A_{ro}/A_{oo}) for the various damage cases on the ultimate longitudinal strength of the ship (M_u/M_{uo}), as shown by equation 2:

$$\alpha = \theta_{IB} / \theta_{OB} \tag{2}$$

where θ_{IB} is slope of the curve of the ultimate longitudinal strength (M_u/M_{uo}) versus the amount of grounding damage for the inner bottom (A_{ri}/A_{oi}), θ_{OB} is slope of the curve of the ultimate longitudinal strength (M_u/M_{uo}) versus the amount of grounding damage for the outer bottom (A_{ro}/A_{oo}). Definition of the above mentioned slopes is shown in [5].

In this paper, the values for correction factor α are taken from [6], which are calculated by the ALPS/HULL Intelligent Supersize Finite Element Method (ISFEM) for aframax tanker. For hogging condition the slopes of the curves for inner and outer bottom are $\theta_{IB} = -0.189$ and $\theta_{OB} = -0.253$ respectively. Therefore the correction factor reads $\alpha=0.747$. For sagging condition the slopes of the curves for inner and outer bottom are $\theta_{IB} = -0.057$ and $\theta_{OB} = -0.174$ respectively. Therefore the correction factor reads $\alpha = 0.3276$.

In order to calculate *GDI* by Equation 1, it is necessary to estimate damage extent in the outer and inner bottom plating. Damage breadth in the outer bottom is calculated from random variable x_3 ($r_1 = x_3 * B$), while damage in the inner bottom (r_2) is calculated for assumed conical shape of the rock (Figure 4), applying the following expression:

$$r_2 = r_1 - 2h_{DB} \tan \frac{\phi}{2} \tag{3}$$

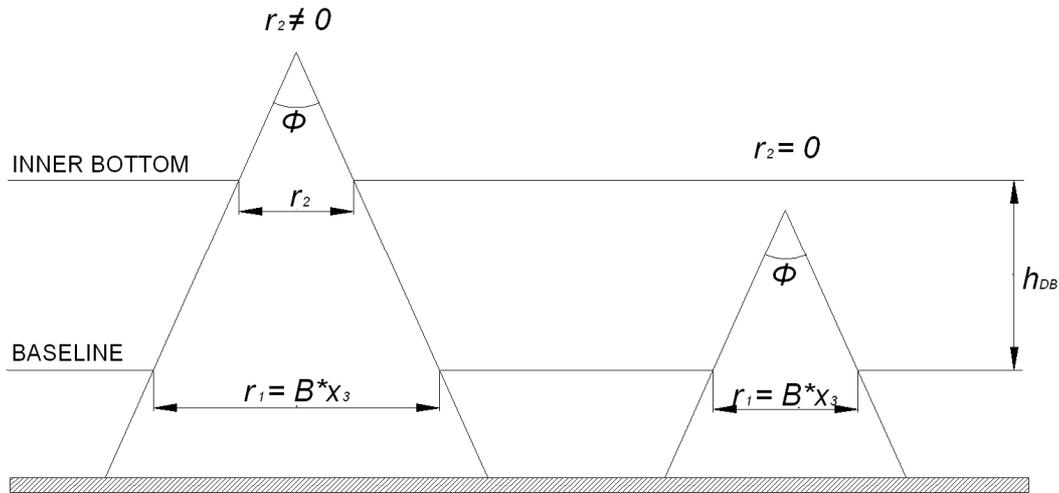


Fig. 4. Conical shape of the rock for cases when double bottom is penetrated (left) and without penetration (right)

The result of the above described calculation is one curve of common influence of damaged inner and outer bottom structures on the ultimate longitudinal strength (M_u/M_{uo} - GDI) instead of the separate curves of dependency between the ultimate longitudinal strength (M_u/M_{uo}) and double-bottom structures (A_{ri}/A_{oi} and A_{ro}/A_{oo}) for the various damage cases. Regression equations using GDI as a main parameter (equations 4 and 5) are derived from M_u/M_{uo} - GDI dependency as [5]:

$$\text{For hogging: } M_u/M_{uo} = -0.0036GDI^2 - 0.3072GDI + 1.0 \quad (4)$$

$$\text{For sagging: } M_u/M_{uo} = -0.1941GDI^2 - 0.1476GDI + 1.0 \quad (5)$$

In the case that outcome of the random variable x_2 multiplied by D exceeds the height of the double bottom, random variable x_4 is generated representing angle Φ of the rock. Generally, variable Φ is between 15° and 150° [6]. However, in order to get credible results, it is also necessary to limit maximum value of Φ for each damage scenario given by combination of damage breadth x_3*B at the outer bottom and damage height x_2*D :

$$\Phi_{max} = 2 \tan^{-1} \frac{x_3*B}{2x_2*D} \quad (6)$$

In such a way, some kind of correlation is established between damage parameters and the rock that caused such damage. In case that Φ would be independent on damage parameters, unrealistic results are produced, as e.g. very high and narrow damage are caused by wide rock that is obviously unrealistic.

It is further assumed that rocking angle is normally distributed random variable with mean value $\Phi_{mean}=(15+\Phi_{max})/2$, and standard deviation $\sigma = (\Phi_{max} - \Phi_{mean})/2$. Thus, maximum value is 2 standard deviations away from the mean rocking angle used in each simulation.

Another dependency is introduced in cases when half of damage breadth ($x_3*B/2$) added to the transverse damage location (x_1*B) exceeds half of the ship breadth. In such cases, damage breadth is reduced to the maximum permissible value.

2.3. Monte Carlo (MC) simulation

Using principal dimensions together with the IMO's probability density distributions and assumptions of the rock's shape, grounding damage parameters may be simulated by the MC simulation method. Variables x_1 , x_2 and x_3 are drawn from their corresponding probability distributions F_1 , F_2 and F_3 . After that, random rocking angle is drawn from the normal distribution as described in the previous paragraph. For each outcome of the random damage scenario, grounding damage index GDI is calculated by equation (1) and then residual ultimate bending moment by equations (4) and (5).

The actual double-bottom height depends on the transverse position of the damage, i.e. it takes into account sloped longitudinal bulkhead and the lowest stringer of the double hull. Definition of actual double-bottom height (h_{DB}) at the transverse damage position is given by equations (7).

1000 MC-simulated grounding damage scenarios are calculated. For the known CDFs denoted $F(x)$ from the interval 0-1 the appropriate damage parameters values are calculated as the inverse transformation $x=F^{-1}$.

Calculation procedure can be summarised as follows:

1. Simulation of the transverse damage location (x_1) from expressions relating curves in Figure 1
2. Simulation of the damage height extent (x_2) from expressions relating curves in Figure 2
3. Simulation of the transverse damage extent (x_3) from expressions relating curves in Figure 3
4. Reduction of the transverse damage extent x_3 is formulated as follows:

We introduce the condition: $B \cdot x_3 / 2 > B/2 - B \cdot x_1$

- Damage extent if the condition is fulfilled, i.e. damage is outside of the ship:

$$B/2 - B \cdot x_1$$

- Damage extent if the condition is not fulfilled: $B \cdot x_3 / 2$

5. Simulation of the assumed angle of the rock (x_4) according to the normal distribution with following parameters:

$$\Phi_{max} = 2 \tan^{-1} \frac{B x_3}{2 D x_2}; \quad 15 \leq \Phi_{max} \leq 150$$

$$\Phi_{mean} = \frac{15 + \Phi_{max}}{2}; \quad \sigma = \frac{\Phi_{max} - \Phi_{mean}}{2}$$

6. Checking if the actual double-bottom height is penetrated. Calculation of r_2 according to equation (3), A_{ro}/A_{oo} and A_{ri}/A_{oi} .
7. Calculation of grounding damage index (GDI) according to equation (1).
8. Calculation of residual strength for damaged ship in hogging and sagging condition by applying regression Equations (4) and (5).
9. Steps 1-8 have been repeated $N=1000$ times.
10. Interpretation of the results for 1000 random generated variables i.e. damage cases using probability density functions.
11. Fitting of the exponential and 2-parameter Weibull distribution functions to the MC probability density function.

3. Case study

The presented simulation procedure is applied to calculate probability distribution of the residual longitudinal strength of the Aframax oil tanker. Firstly, histogram of residual strength is

determined and appropriate probability distribution function is then fitted to that histogram. Then, influence of some design modification on the probability distribution is investigated.

3.1. Description of oil tanker

Main particulars of double-hull oil tanker analysed in the present study are shown in Table 1.

Table 1. Main particulars of oil tanker

Dimension	Unit (m, dwt)
Length between perp., Lpp	234
Breadth, B	42
Depth, D	20
Draught, T	14
Deadweight, DWT	105000
Double bottom height, hDB	2.3
Breadth of inner bottom, bDB	16.4
Position of inner shell from C	18.95

Definition of actual double-bottom height (h_{DB}) at the transverse damage position is given by the following expressions:

$$h_{DB} = 2.3 \text{ m (from CL to 16.4 m)}$$

$$h_{DB} = 5.3 \text{ m (from 18.95 m to 21 m i.e. from inner to outer shell)} \quad (7)$$

$$h_{DB} = 2.3 + \frac{2.3-5.3}{16.4-18.95} (x_1 \cdot B - 16.4), \text{ from side girder to inner hull.}$$

Midship section of the oil tanker with grounding damage definition is shown in Figure 5.

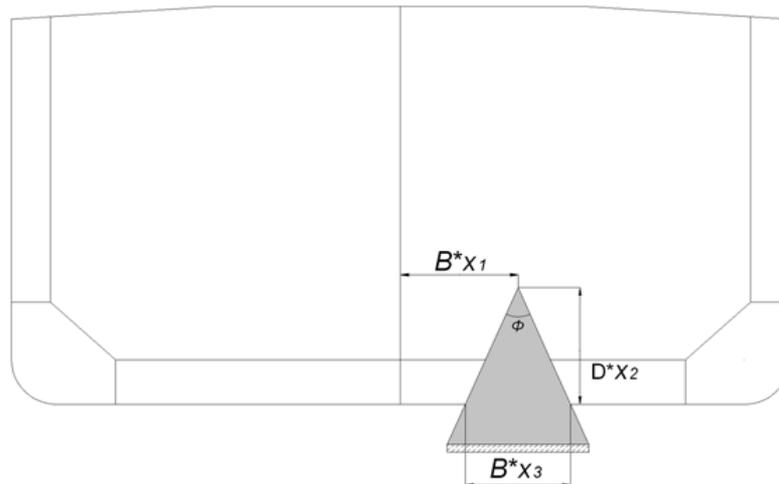


Fig. 5. Location and extent of grounding damage

3.2. Results of the analysis

Percentage of loss of the ultimate strength $(1 - M_u/M_{uo}) \times 100$ for hogging and sagging condition are presented in form of histograms (Figures 6 and 7). Based on the shape of histograms, either exponential or 2-parameter Weibull distributions are considered as good candidates to approximate histograms by theoretical probability function. Method of moments is employed to fit theoretical

distributions. Thus, Weibull distribution is fitted by matching average value and standard deviation calculated from histograms. Probability density function and cumulative distribution functions of Weibull distribution are given as, respectively:

$$f(x) = \frac{k}{\lambda} \cdot \left(\frac{x}{\lambda}\right)^{k-1} \cdot e^{-\left(\frac{x}{\lambda}\right)^k} \quad F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^k} \quad (8)$$

where k, λ are shape and scale parameters of the Weibull probability distribution function, while x is the random variable representing percentage of the loss of the ultimate strength, i.e. $x = (1 - M_u/M_{uo}) \times 100$. It should be noted that exponential distribution is the special case of the Weibull distribution, with exponent $k=1$.

Mean value μ and variance σ^2 of the Weibull distribution are given as [7]:

$$\mu = \frac{1}{\lambda} \Gamma\left(1 + \frac{1}{k}\right) \quad \sigma^2 = \frac{1}{\lambda^2} \Gamma\left(1 + \frac{2}{k}\right) - \mu^2 \quad (9)$$

Gamma function, $\Gamma(2)=1$ and $\Gamma(3)=2$, so obviously for exponential distribution mean value and standard deviation are the same and read $\mu = \sigma = 1/\lambda$. Thus, parameter λ of the exponential distribution is determined from average value \bar{x} of the random variable x calculated from the histogram as $\lambda = 1/\bar{x}$.

Fitting of the Weibull distribution is not so simple, as there are 2 unknown parameters k and λ that can not be directly determined. The procedure of calculating these two parameters requires their optimization in order that mean value \bar{x} and variance σ_x^2 calculated from simulated histogram become equal to the mean value and variance of the Weibull distribution given by equations (9). The procedure may be efficiently performed using Solver option in MS Excell.

Theoretical distribution fitted to histograms are presented in Figures 6 and 7 for hogging and sagging respectively. As it is not obvious from figures which of two theoretical distributions represent better approximation, the analysis using χ^2 -test is performed. Conclusion is that the Weibull function represent better estimate for hogging, while exponential function is more suitable for sagging.

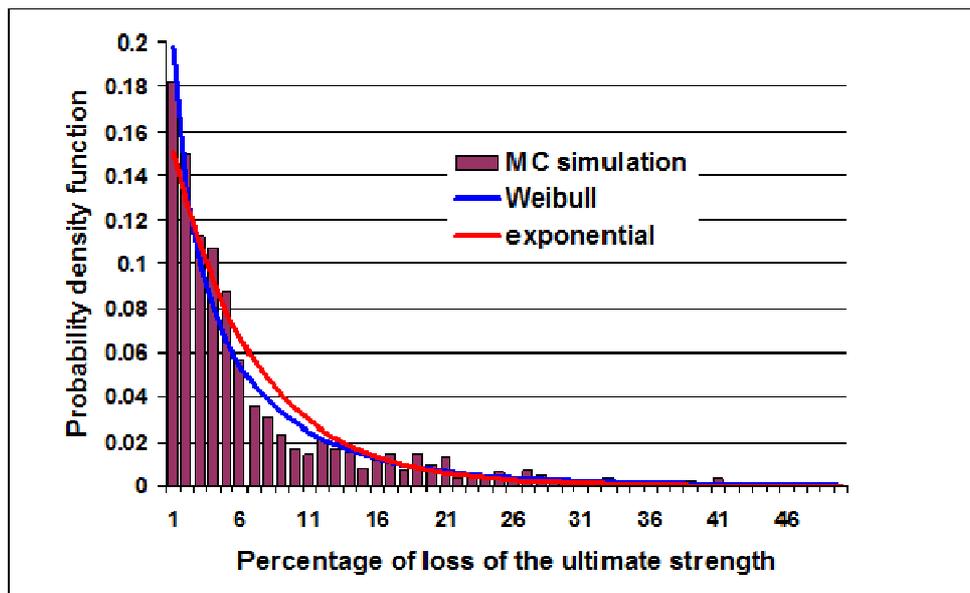


Fig. 6. Histogram of loss of the ultimate strength for hogging condition with fitted Weibull function;

$$\bar{x} = 6.13, k = 0.809, \lambda = 5.624$$

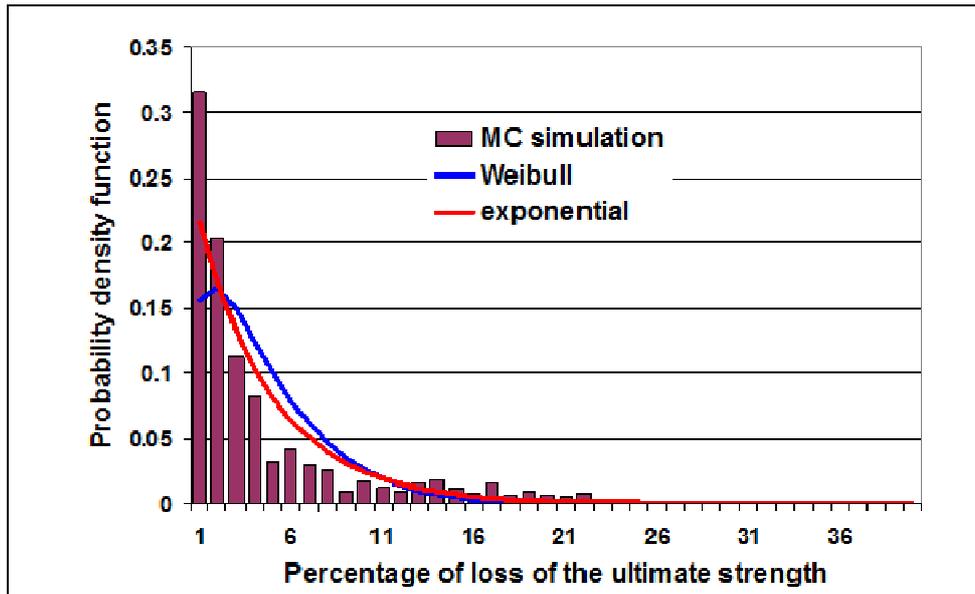


Fig. 7. Histogram of loss of the ultimate strength for sagging condition with fitted exponential function;

$$\bar{x} = 4.127, k = 1.232, \lambda = 4.445$$

Statistical analysis of the procedure error is made by calculating mean values \bar{x} of the percentage of the loss of intact bending moment capacity. The calculation is performed by repeating described simulations 30 times with different sets of random numbers. For such a sample of $n=30$ MC-simulated arithmetic values, confidence interval is then calculated.

Results of such statistical analysis indicate that 1000 simulations performed in the present study provide accurate estimate of the mean value, as the “true” mean value with 95.45% confidence lies in the interval whose width is only +/-1.5% from the expected mean $\mu = 6.13$ in hogging and +/-1.7% from the expected mean $\mu = 4.127$ in sagging. Therefore, increased number of MC simulations will not affect results of the study.

4. Applications

The presented analysis may have several useful applications. It may be used in evaluating consequences of the design modifications on the residual ultimate strength of grounded ships, as shown in the following section 4.1. It also may be used in structural reliability studies of damaged ships, as proposed in the section 4.2.

4.1. Influence of design modifications

The same calculation is performed four times more, in the cases when the double-bottom height (h_{DB}) and the breadth of inner double-bottom shell (b_{DB}) differ from original design value +10% and -10%.

The average values of the losses of the ultimate strength for hogging and sagging condition are shown in Table 2. Obviously, modification of the double-bottom height and breadth has minor influence on the ultimate strength loss. Of course, there is influence of those parameters on intact ultimate strength and consequently on the ship structural safety that is not considered in the present study. The intention of this chapter was only to show how much the double-bottom height influence on the loss of the ultimate bending moment in damaged condition.

Table 2. Influence of structural modifications on the loss of the ultimate strength (average values in %)

Modified imension	Hogging	Sagging
b_{DB}/h_{DB}	6.130	4.127
$(b_{DB}+10\%)/h_{DB}$	6.195	4.149
$(b_{DB}-10\%)/h_{DB}$	6.131	4.125
$b_{DB}/(h_{DB}+10\%)$	6.053	4.118
$b_{DB}/(h_{DB}-10\%)$	6.258	4.147

4.2. Application in structural reliability assessment of grounded ships

Structural reliability of grounded ships is recently studied by [3],[8],**Error! Reference source not found.** The commonly used limit state function for the failure of the damaged hull girder under vertical bending moment reads:

$$g = \hat{\chi}_u RIF M_u - (K_{US} \hat{\chi}_{sw} M_{sw} + K_{UW} \hat{\chi}_w \hat{\chi}_{nl} M_w) \quad (10)$$

where $\hat{\chi}_u$, $\hat{\chi}_w$ and $\hat{\chi}_{nl}$ are the modelling uncertainties of the ultimate strength, linear wave load effects and non-linear wave load effects, respectively, K_{us} and K_{uw} are the load combination coefficients for the still-water and wave loads, respectively, M_u is the ultimate vertical bending moment, M_{sw} is the still-water vertical bending moment, M_w is the wave bending moment and RIF is the residual strength index. The latest variable, RIF , is defined as the ratio of the ultimate moment of the damaged section and the ultimate moment of the intact section [8]. Therefore, the meaning of the RIF is exactly the same as given by equations 4 and 5. In most of the studies, RIF is considered as deterministic quantity, subjected to the parametric studies that takes into account various damage extends. However, presently developed probabilistic RIF represents the rational estimate of the damage consequences and may be used directly in the equation (5). That represents the contribution of the presented study to the state-of-the-art in the structural reliability assessment of grounded oil tanker.

5. Conclusion

Probabilistic model of the ultimate strength of grounded ship is developed based on the MC simulation. The probabilistic model for transverse location and size of the damage is prescribed by IMO. It is assumed that grounding is caused by the rock of the conical shape. That assumption is used to correlate damage of the outer and inner bottom. The angle of the rock is considered as random value limited by the height and breadth of the grounding damage. Based on the grounding damage index (GDI), ultimate strength of the damaged ship is calculated by the regression equation developed in [6].

The outcome of MC simulations is the histogram showing that the most of damages cause fairly low reduction of the ultimate strength of the intact ship. Frequency of large losses of ultimate strength is reducing relatively fast. Based on this characteristics, it seems that histogram may be reasonably approximated by either the exponential distribution either 2- parameter Weibull distribution. The average loss (in percentage) for the aframax tanker reads about 6.3% and 4.2% in hogging and sagging respectively.

The main purpose of this study is development of the probabilistic model for the bending capacity of the damaged ship to be used in the structural reliability studies within the scope of the safety of the maritime transportation. However, results may also be used in the assessment of consequences of structural modifications on the ultimate strength of damaged ship. It is

demonstrated in the paper that increasing or decreasing height or breadth of the double bottom cause only minor modifications of the ultimate longitudinal strength loss in the case of grounding accident.

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