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COORDINATE TRANSFORMATION IN NANOROBOTICS

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ABSTRACT
As it is the well known, the nanorobotics is the multidisciplinary field that deals with the controlled manipulation of atomic and molecular-sized objects and therefore sometimes is called molecular robotics. In order to design control algorithm for control of a nanorobot motion in a multipotential field, one should start with the related coordinate transformations. It has been done in this paper for two nanorobots, or two nanoobjects generally, where first one is moving relative to the second one in an alpha field with a velocity v_α, along the positive axis of x. Here an alpha field denotes a multipotential field that influents to the nanorobot motions. The obtained relativistic coordinate transformation model for two nanorobots has also been reduced to the nonrelativistic case of the nanorobots motions.
Keywords: nanorobots, coordinate transformation, multipotential field

1. INTRODUCTION
The state of the art in nanorobotics has been presented in the references [1,2,3,4]. As it is the well known, the nanorobotics is the multidisciplinary field that deals with the controlled manipulation of atomic and molecular-sized objects and therefore sometimes is called molecular robotics [1]. Generally, there are two main approaches for building useful devices from nanoscale components. The first one is based on self-assembly, and is a natural evolution of traditional chemistry and bulk processing [3]. The second approach is based on control of the positions and velocities of nanoscale objects by direct application of mechanical forces, electromagnetic fields, and the other potential fields. The research in nanorobotics in the second approach has proceeded along two lines. The first one is devoted to the design and computational simulation of robots with nanoscale dimensions [4]. The second line of nanorobotics research involves manipulation of nanoscale objects with macroscopic instruments and related potential fields like Scanning Probe Microscopy (SPM), Scanning Tunneling Microscope (STM), and Atomic Force Microscope (AFM). The spatial region in nanorobotics is the bionanorobotics [5]. Potential applications of the nanorobots are expected in the tree important regions: nanomedicine, nanotechnology and space applications.
In order to control nanorobots in mechanics, electronics, electromagnetic, gravitomagnetic, photonics, chemical and biomaterials regions we have to have the ability to construct the related artificial control potential fields [2]. At the nanoscale the control dynamics is very complex because there are very strong interaction between nanorobots and nanoenvironment. Thus, the first step in designing of the control dynamics for nanorobots is the development of the relativistic coordinate transformations including the all interactions in a multipotential field. It has been done in this paper for two nanorobots, or two objects generally, where the first one is moving relative to the second one in an alpha field with a velocity v_α, along the positive axis of x. Here an alpha field denotes a multipotential

field that influents to the nanorobots motions. The obtained relativistic coordinate transformation model for two nanorobots has also been reduced to the nonrelativistic case of the nanorobots motions.

2. DESIGN OF COORDINATE TRANSFORMATIONS IN AN ALPHA FIELD USING GROUP POSTULATES

The term an Alpha Field (AF) denotes any potential field that can be described by two dimensionless field parameters α and α' [2,10]. Generally, the field parameters α and α' are functions of the space-tame coordinates in the related potential field. For an example, if a nanorobot is moving in a gravitational field, then the field parameters α and α' are functions of the space-tame coordinates in a gravitational field, and should be determined by solution of the Einstein's field equations. On the other hand, if there is no potential field (i.e. a nanorobot is moving in a vacuum) then the field parameters α and α' are constants and satisfy the relation $\alpha = \alpha' = 1$. Thus, the field parameters α and α' should be determined in each particular potential field by solution of the related field equations.

As it is well known, if space-time is homogeneous, then the coordinate transformations (Lorentz transformations) must be linear transformations [6,7,8,9]. This implies that the relative velocity v_α between the reference frames K of the first nanorobot and K' of the second nanorobots in an alpha field must be constant. In that case we have inertial reference frames K and K'. Generally, for a relative motion of the nanorobots reference frames K and K' in an alpha field, the relative velocity v_α is a composition of the two velocities (v and v_f). Here v is a velocity of the nanorobot reference frame K' relative to the nanorobot reference frame K in a vacuum, without any potential field (a free nanorobots motions). The velocity v_f shows an influence of an alpha field to the motion of a nanorobot reference frame K' relative to the nanorobot reference frame K in that field. For an example, v_f could be a free fall nanorobot velocity in a gravitational field. Therefore, the velocity v_f should be a function of the field parameters α and α'. Taking into account the previous consideration, the velocity v_α can be calculated by the relation:

$$\begin{gathered} v_\alpha = v_{\alpha_x} = v_x + v_{f_x} = v\cos\varphi + v_f\cos\psi = v_x - \frac{\kappa(\alpha-\alpha')_x c}{2}, \\ \varphi = \psi = 0 \quad \rightarrow \quad v_\alpha = v_{\alpha_x} = v - \frac{\kappa(\alpha-\alpha')c}{2}. \end{gathered} \tag{1}$$

Here φ and ψ are angels between vectors ($\vec{v}, \vec{v}_\alpha$) and ($\vec{v}_f, \vec{v}_\alpha$), respectively. In the relation (1) we assume that the observation signal is the light with invariant velocity c in both nanorobots reference frames K, and K'. Finally, we can employ, for the convenience, an observation parameter κ. Thus, one can put $\kappa = 1$ if an observation signal is emitted from the origin of the nanorobot reference frame K, or $\kappa = -1$ if an observation signal is emitted from the origin of the nanorobot reference frame K'.

Generally speaking, for motion in an alpha field a relative velocity v_α between nanorobot reference frames K' and K is not a constant. In order to derive a linear coordinate transformations model, one should assume that in the infinitesimally small space-time regions of an alpha field (dx, dt) and (dx', dt'), a relative velocity v_α is a constant. In that case the coordinate transformations in an alpha field, from the nanorobot inertial frame K to the nanorobot inertial frame K', transform a linear motion in (dt, dx) into a linear motion in (dt', dx') coordinate system. This is a local Lorentz transformation.

As it is the well known, the coordinate transformations between inertial frames form a group [8]. This group is called the proper Lorentz group with the group operation being the composition of transformations. This means performing one transformation after another. In that sense, the following four group axioms should be satisfied [8]. a) Closure: the composition of two transformations is a transformation. In such a manner a composition of transformations from the inertial frame K to inertial frame K' and then from K' to inertial frame K'' can be replace with a transformation directly from an inertial frame K to inertial frame K'':

$$[K \rightarrow K'][K' \rightarrow K''] = [K \rightarrow K'']. \tag{2}$$

b) Associativity: the result of the following two transformations is apparently the same:

$$\left(\left[K \to K'\right]\left[K' \to K''\right]\right)\left[K'' \to K'''\right] = \left[K \to K'''\right],$$
$$\left[K \to K'\right]\left(\left[K' \to K''\right]\left[K'' \to K'''\right]\right) = \left[K \to K'''\right]. \tag{3}$$

c) Identity element: there is an identity element, a transformation K $\to$ K. d) Inverse element: for any transformation K $\to$ K' there apparently exists an inverse transformation K' $\to$ K.

Now, let a nanorobot inertial frame K' is moving in an alpha field with a velocity $\vec{v}_\alpha$ relative to a nanorobot inertial frame K. Using rotations and shifts operations one can choose the x axis in K and x' axis in K' along the relative velocity vector $\vec{v}_\alpha$ and that the events (t = 0, x = 0) and (t' = 0, x' = 0) coincide. The velocity boost is along the x and x' axes only, therefore nothing happens to the perpendicular coordinates (y, z) and (y', z') and one can just omit them for brevity. The transformation [K$\to$K'] connects two nanorobot inertial frames. Therefore it has to transform a linear motion in (t, x) into a linear motion in (t', x') coordinates. The conclusion is that the transformation [K$\to$K'] must be a linear transformation. This also includes that a relative velocity v_α between nanorobot reference frames K and K' should be a constant. Meanwhile, as it is presented previously, a motion in an alpha field with relative velocity v_α from (1), between nanorobots reference frames K' and K, generally is not a constant. In order to derive a linear coordinate transformation model, one should assume that in the infinitesimally small space-time regions of an alpha field (dx, dt) and (dx', dt'), a relative velocity v_α is a constant. In that case the coordinate transformations in an alpha field [K$\to$K'] transforms a linear motion in (dt, dx) into a linear motion in (dt', dx') coordinate system.

Let the general form of the linear transformations in an alpha field is given by a matrix form:

$$\begin{bmatrix} dt' \\ dx' \end{bmatrix} = \begin{bmatrix} N & M \\ L & H \end{bmatrix} \begin{bmatrix} dt \\ dx \end{bmatrix}. \tag{4}$$

Here parameters N, M, L and H are some, yet unknown, functions of the relative velocity v_α. The origin of the nanorobot reference frame K' has coordinate differentials (dt', dx' = 0) in that frame, while in the nanorobot reference frame K it has coordinate differentials (dt, dx = $v_\alpha$dt). These two points are connected by the coordinate transformation in (4):

$$\begin{bmatrix} dt' \\ 0 \end{bmatrix} = \begin{bmatrix} N & M \\ L & H \end{bmatrix} \begin{bmatrix} dt \\ v_\alpha dt \end{bmatrix}, \quad \to \quad L = -v_\alpha H. \tag{5}$$

Analogously, considering the motion of the origin of the frame K in an alpha field, one obtains:

$$\begin{bmatrix} dt' \\ -v_\alpha dt' \end{bmatrix} = \begin{bmatrix} N & M \\ L & H \end{bmatrix} \begin{bmatrix} dt \\ 0 \end{bmatrix}, \quad \to \quad L = -v_\alpha N. \tag{6}$$

Comparing the relation (5) with the relation (6) one can conclude that N = H. This enables a simplification of the transformation matrix to the form:

$$\begin{bmatrix} dt' \\ dx' \end{bmatrix} = \begin{bmatrix} H & M \\ -v_\alpha H & H \end{bmatrix} \begin{bmatrix} dt \\ dx \end{bmatrix}. \tag{7}$$

Following the considerations of the fourth group axioms one can find out the unknown parameters H and M. Further, applying the procedure from the reference [8], one obtains the coordinate transformation model in nanorobotics, valid in an alpha field:

$$\begin{bmatrix} dt' \\ dx' \end{bmatrix} = H \begin{bmatrix} 1 & \dfrac{-v_\alpha}{c^2} \\ -v_\alpha & 1 \end{bmatrix} \begin{bmatrix} dt \\ dx \end{bmatrix}, \quad H = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2}. \tag{8}$$

Now, one can include the structure of the relative velocity v_α between nanorobots reference frames K' and K along the positive axis of x given by (1). In that case, the coordinate transformation model in nanorobotics from (8) is transformed into the final relations:

$$\begin{bmatrix} dt' \\ dx' \end{bmatrix} = H \begin{bmatrix} 1 & -\left(\frac{v_x}{c^2} - \frac{\kappa(\alpha - \alpha')_x c}{2c^2} \right) \\ -\left(v_x - \frac{\kappa(\alpha - \alpha')_x c}{2} \right) & 1 \end{bmatrix} \begin{bmatrix} dt \\ dx \end{bmatrix}, \tag{9}$$

$$H = \left(1 - \frac{v_\alpha^2}{c^2} \right)^{-1/2} = \left(\alpha\alpha'_x - \frac{v_x^2}{c^2} + \frac{\kappa(\alpha - \alpha')_x c v_x}{c^2} \right)^{-1/2} .$$

If the motion is in a vacuum, without any potential field, then the field parameters α and α' satisfy the relation $\alpha = \alpha' = 1$ and $v_x = v$. In that case and for v = const. the model (9) is transformed into the well known Lorentz transformation model [6,7,8,9,10]. On the other side, for $v_\alpha << c$ we have H = 1, and models (8) and (9) are transformed into the nonrelativistic coordinate transformation model valid in nanorobotics. Calculation of the parameters α and α′ are presented in the references [2,10].

3. CONCLUSION

In this paper it has been presented the relativistic coordinate transformations in a multipotential field that we usually have in the nanorobotics. The obtained coordinate transformations model is function on the dimensionless field parameter α and α′. These parameters include the influences of the multipotential field to the nanorobots motion in it. Generally, a relative velocity v_α between two nanorobots, or two objects in a multipotential field is not a constant. Therefore, the coordinate transformation model is designed in a differential form. Thus, the reference frames in the presented coordinate transformations model are local inertial frames. In the case where the relative velocity v_α is a constant, the obtained model is transformed into the well known Lorentz transformation model. Finally, if $v_\alpha << c$, then the presented model is transformed into the nonrelativistic one, valid for the nonrelativistic motions of two nanorobots, or two nanoobjects, generally, in the multipotential field.

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